

1990

# Probabilistic Analysis of Bridge Scour Using the Principle of Maximum Entropy.

Donald Edward Barbe'

*Louisiana State University and Agricultural & Mechanical College*

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maximum entropy**

**Barbe', Donald Edward, Ph.D.**

**The Louisiana State University and Agricultural and Mechanical Col., 1990**

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Ann Arbor, MI 48106



**PROBABILISTIC ANALYSIS OF BRIDGE SCOUR  
USING THE PRINCIPLE OF MAXIMUM ENTROPY**

**A Dissertation**

**Submitted to the Graduate Faculty of the  
Louisiana State University and  
Agricultural and Mechanical College  
in partial fulfillment of the  
requirements for the degree of  
Doctor of Philosophy**

**in**

**The Department of Civil Engineering**

**by**

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### **ABSTRACT**

The probability distribution function for floods and velocity profile in open channel flow are combined with scour theory to present a procedure for determining the risk of bridge failure due to scour. This is accomplished by deriving a probability distribution function for scour at a bridge pier.

The theory of entropy and the principle of maximum entropy are used to determine the velocity profile. The constraints used in the entropy maximization are derived from the physical laws of conservation of mass, momentum, and energy. A procedure to approximate the solution to the resulting equations is shown.

The vertical velocity is used to determine an effective depth of flow. The specific energy of the flow within this effective depth is used to develop a scour estimation procedure for use at a bridge pier.

The probability distribution function for scour at a bridge pier is obtained by applying the procedure for each level of discharge in the flood probability distribution function. The procedure is applied at a pier and the results compared to the actual field measurements.

## CHAPTER 1

### INTRODUCTION

Bridge failure due to scour has long been a problem for civilization. During the Ottoman Empire of the middle ages attempts were made to determine the maximum scour around bridge piers (Simons and Senturk, 1976). As has been seen by recent bridge failures on the interstate highway system, the "best" highway systems are not immune to this problem.

The Federal Highway Administration has estimated that the cost of bridge damage due to floods is in the millions of dollars annually (Hopkins, et al, 1979). This is the cost of the physical damage to the bridge structures themselves. The additional loss to society of property, commerce, and time is difficult to estimate. Also the tragic loss of life, as has happened recently, cannot be measured in monetary terms.

Society's search for a reliable method to estimate the scour at a bridge pier has been a long and sometimes fruitless task. Galileo Galilei (1564-1642), the noted astronomer, also investigated the movement of water. He stated that the precise movement of the stars would be discovered one day but the laws governing the flow in a simple brook may take much more time to be understood (Simons and Senturk, 1976).

The problem of sediment erosion and transport is one of the most difficult problems in hydraulic engineering. In spite of its importance, there exists a great difference between the information needed and the information available on this subject. The variables present in the scour process are the phenomena of non-uniformity, unsteadiness in the flow, ordinary turbulence and macroturbulence, waves, moving beds, constantly changing channel characteristics, and the flow of mixtures of water and solid particles (Morris and Wiggert, 1972).

The natural phenomenon of sediment transport can be greatly increased in rivers with a non-cohesive bed material because of manmade "improvements". Manmade structures such as bridges cause constriction of the natural flow. This in turn causes an increase in the local velocity at the bridge pier. The increased local velocity contributes to scour.

Henderson (1966) has shown that the sediment specific discharge is proportional to the square of the total flow specific discharge. This implies that the greatest flood flows are responsible for the majority of the scour annually. Therefore, the greatest scour occurs over the short duration of the maximum floods. Bridges can therefore become in greater risk of failure in a relatively short time.



As increased urbanization and decreased available natural flood plains cause a greater number of floods, the number of bridge failures due to scour can be expected to increase. In Louisiana at present, scour problems exist and are being monitored at approximately 80 sites (Strautmann, 1987).

As history has shown, most advances in river hydraulics have taken place as the result of specific problems that require immediate solution. It is to this end that this research attempts to investigate this problem, more from the stochastic than from the deterministic approach. The myriad of scour formulas that have been presented in the past have been based on flume experiments and they yield very different results. This research will attempt to produce a rational approach to the evaluation of the bridge scour problem.

The approach investigated in this study is a procedure to determine a probability distribution for scour. A method to produce the maximum scour corresponding to each flood flow in the flood PDF will be studied. The scour procedure will use the concept of an effective depth. The effective depth is the point in the flow above which the flow is essentially horizontal and therefore does not contribute to scour. A formula for the maximum scour that is related to the velocity profile below the effective depth will be studied.

### 1.1 Objectives

The objective of this research is to develop a framework for a methodology to evaluate the risk of bridge failure due to pier scour during floods. This methodology would be based upon the following tasks:

- 1) Derive a probability distribution of floods at the site.
- 2) Develop a horizontal velocity profile for each flood.
- 3) Develop a method to evaluate the strength of the horseshoe vortex for each flood.
- 4) Relate the horizontal velocity profile and the strength of the horseshoe vortex to the scour.
- 5) Evaluate the new method on observed data.
- 6) Show a sample application on a field case, that is determine the scour PDF.
- 7) Do the structural analysis of the bridge, that is find the hydraulic force and the resisting force for the bridge structure for each flood.

The specific tasks to be completed in this research are task numbers 2 through 6.

The method that will be proposed to evaluate the strength of the horseshoe vortex (Task 3) is to define a depth called

the effective depth. This will be the depth of the approach flow that is influenced by the horseshoe vortex.

The horizontal velocity profile within the effective depth will be used to obtain the specific energy within this zone. The maximum scour will be related to this specific energy (Task 4).

A new method for obtaining the velocity profile will be introduced to be able to more accurately obtain the specific energy within the effective depth (Task 2).

## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 Introduction**

For the task numbered 1 through 4 in the objectives, the following literature review will be undertaken:

- 1) Partial survey of methods for obtaining probability distribution functions for floods.
- 2) Survey methods for obtaining velocity distributions in open channel flow.
- 3) Survey of local scour.
- 4) Survey of the measurement of the vertical velocity at a bridge pier.

## 2.2 Probability Distribution Function for Floods

Methods which have been developed to estimate a flood probability distribution function, PDF, are usually of two types. The first type is the non-parametric or graphical methods (Beard, 1962) and the second is the more popular parametric approach which allows for the estimation of the uncertainty associated with the PDF.

The parametric approach represents the annual flood series,  $afs$ , by a preselected probability density function (Foster, 1924). Distributions which have been recommended for the parametric approach (Cruise and Arora, 1990) include the log Pearson type III (WRC, 1981), log normal (Chow, 1954), extreme value type I, EVI, (Gumbel, 1941), generalized extreme value (Hosking, et al, 1985), Wakeby (Houghton, 1978) and the two component extreme value distribution (Rossi, et al, 1984).

Recently, stochastic flood models have been developed which obtain estimates of the annual flood distribution by modeling the partial duration series,  $pds$ . The  $pds$  consists of all recorded values above an assumed base level. Langbein (1949) was the first to use the  $pds$  in this manner when he used general probability concepts to determine a relationship between the  $afs$  and the  $pds$  (Cruise and Arora, 1990).

Present research efforts are centered in three areas, parameter estimation, robustness of distributions, and regional distribution methods.

The research presented in this study will not deal with the subject of the derivation of flood probability distribution functions.

## **2.3 Horizontal velocity distribution in open channel flow**

### **2.3.1 Introduction**

The horizontal velocity distribution in a channel must be known for solution of many engineering problems. The velocity distribution or profile is very different for the two types of possible flow, namely, laminar flow and turbulent flow. The flow in open channels of alluvial sand beds is generally hydraulically rough. Therefore turbulent flow prevails for most natural conditions.

We are able to define the velocity at a given time and point in laminar flow. However, in turbulent flow this precision is not possible. In turbulent flow the velocity vector is not constant, it fluctuates both spatially and temporally. Therefore we will deal only with the time averaged velocity in this study.

There are presently two basic methods and a recently proposed method to obtain a time averaged horizontal velocity distribution: the logarithmic distribution law, the power law, and the entropy method. All three methods will be compared in this study.

### 2.3.2 Prandtl - von Karman logarithmic law

The Prandtl - von Karman universal logarithmic velocity distribution law was developed for pipe flow (von Karman, 1935). It is based on two assumptions: (1) that the mixing length is proportional to  $y$  (the depth from the channel bed to the point of interest), and (2) that the shearing stress is constant. Vanoni (1941) showed that this equation could also be applied to a wide open channel. The equation is

$$u = u_D - (u^* / K) \ln(D / y) \quad (2-1)$$

where  $D$  = the depth of flow in the channel,

$u$  = the horizontal velocity at a distance  $y$  from the channel bed,

$K$  = the von Karman universal constant which has a value of 0.40 for clear water and a value as low as 0.2 in flows with heavy sediment loads,

$u_D$  = the maximum velocity of the flow which occurs at the surface of the flow ( $y = D$ ),

and

$u^*$  = the shear velocity.

Equation (2-1) is generally not used in this form. The shear velocity is given by



$$u^* = (g D S)^{1/2} \quad (\text{Chow, 1959})$$

where

$g$  = the acceleration due to gravity,

$S$  = the channel slope.

If equation (2-1) is integrated over the depth and the substitution for the shear velocity is made, an equation in terms of the mean velocity  $u_m$  instead of the maximum velocity is obtained.

$$u = u_m + (u^* / K) ( 1 + \ln(y / D) ) \quad (2-2)$$

This is the Prandtl - von Karman universal logarithmic velocity distribution law for open channel flow.

### 2.3.3 Simple power law

Blasius (1913) found that for Reynolds numbers between 3,000 and 100,000 the velocity profile in a smooth pipe is closely approximated by the expression

$$u / u_D = (y / r_0)^{1/7} \quad (2-3)$$

where  $y = r_0 - r$  is the distance from the pipe wall,

$r_0$  = the radius of the pipe, and

$r$  = the distance from the center of the pipe to the point where the velocity is  $u$ .

This expression is called the seventh root law (Daugherty and Franzini, 1977).

The power law velocity distribution for flow in an open channel (Sarma, et. al., 1983) can be stated as follows:

$$u = u_D (y / D)^{1/n} \quad (2-4)$$

where

$u$  = the velocity at a vertical depth of  $y$  above the channel bed,

$u_D$  = the maximum velocity which occurs at the surface,

$D$  = the total depth of the flow,

and

$n$  = a parameter determined by the frictional resistance at the bed.

Karim and Kennedy (1987) found that  $n$  is usually in the range of 6-7.

Dingman (1989) applied the power law to natural streams to obtain the velocity distribution. He compared his results to the distributions obtained from the application of the Prandtl - von Karman universal logarithmic law and the distribution obtained from the two constraint entropy method of Chiu (1987).

## **2.3.4 Entropy Method**

### **2.3.4.1 Development of the entropy method**

The term entropy was first coined by the German physicist Rudolf Clausius in 1868. It is a consequence of the second law of thermodynamics. Entropy in a closed system is a measure of the amount of energy no longer capable of conversion into work without outside influence. That is, it is a measure of the amount of energy converted from free energy into bound energy. Boltzmann (1872) gave the first mathematical expression of the second law and to the property called entropy.

Shannon (1948a, 1948b) developed a mathematical theory of entropy for application to the field of communications. He used entropy as a measure of the information content of a message sent along a transmission line. This entropy from communications engineering is now regarded as a useful characteristic of any probability distribution.

Jaynes (1957a, 1957b, 1961, 1982) developed the principle of maximum entropy (POME). Here he proposed that the maximization of the entropy, subject to any constraints on the system, is the best means for determining the prior

probabilities, in the least biased manner. That entropy tends to a maximum means that it tends to the state with a maximum number of possibilities of realization, that is, a tendency toward the most probable state. Therefore, if the entropy of a system has a value that is less than its attainable maximum, the system is not in its most probable state and it will likely pass into more probable states.

The concept of entropy provides an excellent way of introducing probability into hydraulic modeling. Entropy can be used as a measure of the degree of uncertainty inherent in random hydraulic processes. It reflects the information content of the space-time measurements of these processes. It is useful in hydraulic modeling and parameter estimation.

Sonuga (1972, 1976) was successful in using POME in frequency analysis and rainfall-runoff relationships. He showed the strengths and limitations of using POME in hydrologic modeling where data are scarce.

Amorocho and Espildara (1973) used the concept of entropy to assess the performance of the Stanford Watershed Model (Crawford and Linsley, 1966). Their work clearly showed the importance and limitations of the concept of entropy in model assessment.

Singh, et al, (1985, 1986) used POME to develop a procedure for derivation of a number of frequency distributions used in hydrology. The distributions derived included the exponential, uniform, normal, 2-parameter lognormal, 3-parameter lognormal, gamma, Pearson type 3, log-Pearson type 3, Gumbel, log-Gumbel, Weibull, extreme value type 3, and beta distributions.

The concept of entropy can be applied in modeling the vertical distribution of the horizontal velocity in open channel flow. Chiu (1987, 1989) used this method to develop a vertical distribution of the horizontal velocity in a wide open channel with uniform flow. Chiu (1988) also derived equations based on the entropy concept for describing the two-dimensional distribution of the horizontal velocity in an open channel cross section.

### 2.3.4.2 The Shannon Entropy Functional

The Shannon Entropy Functional (SEF) was the first mathematical representation of entropy for use in information theory (Shannon, 1948a and 1948b). Consider a probability density function  $f(x)$  for a continuous random variable  $x$ . Then

$$\int_0^{\infty} f(x) dx = 1 \quad (2-5)$$

and  $f(x)$  is positive for all values of  $x$ .

The SEF is defined as

$$I(f) = - \int_0^{\infty} f(x) \ln(f(x)) dx \quad (2-6)$$

$I(f)$  may be thought of as the expected value or mean of  $-\ln(f(x))$ .

The physical meaning of Equation (2-6) can be clarified by using communication theory. Let  $p(x_j|y_j)$  be the conditional probability that a system is in state  $x_j$  after a sample datum  $y_j$  has been taken. Now  $y_j$  denotes the measured datum indicating  $x_j$ , which may include an error such that the

true state is  $x_j + e_j$ , where  $e_j$  is the error in  $y_j$ . Also let  $p(x_j)$  be the probability that the system is in state  $x_j$  without the sample being taken. In information theory, the information content of data is defined as

$$\ln \{ p(x_j|y_j) / p(x_j) \} \quad (2-7)$$

The greater the value of  $\{ p(x_j|y_j) / p(x_j) \}$ , the greater the information content of the data. If the measurement of the data were perfect, such that the measurement error is zero, then  $p(x_j|y_j) = 1$ . The information in the data as shown by Equation (2-7) becomes  $-\ln p(x_j)$ . Thus, the entropy that was defined by Shannon in Equation (2-6) is a measure of the average information content per sample datum.



#### 2.3.4.3 The Principle of Maximum Entropy

The principle of maximum entropy (POME) was formulated by Jaynes (1961, 1962). With POME, the least biased  $f(x)$  is obtained by maximization of the entropy subject to the given constraints (information). The entropy of a system is a maximum when the greatest information about the state variable  $x$  is obtained from the data.

As stated previously, that entropy tends to a maximum means that it tends to the state with a maximum number of possibilities of realization, that is, a tendency toward the most probable state. Therefore maximum entropy should be obtained when  $x$  has a uniform probability distribution.

In real systems, the probability distribution is usually not uniform because of various constraints on the system. In open channel flow, the normal constraints on the system are conservation of mass, energy and momentum. Therefore, entropy measures how close an a priori probability distribution is to the uniform distribution. Maximizing the entropy of a system will make the probability distribution as uniform as possible while satisfying the constraints on the system. Therefore, the laws of probability that govern a system and thus the magnitude of entropy depend on the constraints on the system.

We will use the principle of maximum entropy by maximizing the SEF for our system subject to the real world constraints that apply. This is done as follows:

If we have  $m$  linearly independent constraints  $C_i$  in the form

$$C_i(f) = \int_0^{\infty} g_i(x) f(x) dx, \quad i = 1, 2, \dots, m \quad (2-8)$$

where the  $g_i$ 's are functions of  $x$  whose averages over  $f(x)$  are specified, then the maximum of  $I(f)$  in Equation (2-6) subject to the constraints  $C_i$  of Equation (2-8) is obtained by the method of undetermined parameters. This occurs when

$$\partial R(f) / \partial(f) + \sum_{i=1}^m a_i \partial C_i(f) / \partial(f) = 0 \quad (2-9)$$

where  $a_i$ ,  $i = 1, \dots, m$ , are the Lagrange multipliers and the entropy is given by

$$I(f) = \int_0^{\infty} R(f(x)) dx$$

#### 2.3.4.4 Velocity Distribution by Chiu

From boundary shear considerations, the classical method of describing the velocity profile (von Karman, 1935) is by relating it to the depth. In open channel flow with depth  $D$ , the velocity monotonically increases from zero at the bed, because of maximum boundary shear at the bed, to a maximum value at the surface, because of minimum boundary shear when the water-air interface is neglected. Let  $u$  be the velocity at a distance  $y$  above the channel bed. Then, the probability of the velocity being less than or equal to  $u$  is  $y/D$  and the cumulative distribution function is

$$F(u) = y / D \quad (2-10)$$

and the probability density function is

$$f(u) = (1 / D) (dy / du) \quad (2-11)$$

Following the development of Chiu (1986, 1988), the constraints on the system are based on the three conservation principles, namely conservation of mass, momentum, and energy. These constraints can be derived from the three basic conservation equations for open channel flow.

The equation of conservation of mass in open channel flow is

$$q = \int_0^D u \, dy = u_m D \quad (2-12)$$

where  $q$  = the specific discharge (discharge per unit width), and  $u_m$  = the mean velocity (depth averaged).

Substitution of  $dy$  from Equation (2-11) gives

$$\int_0^{u_D} u D f(u) \, du = u_m D$$

or

$$\int_0^{u_D} u f(u) \, du = u_m \quad (2-13)$$

The equation for conservation of momentum in open channel flow is given by

$$\rho \int_0^D u^2 \, dy = M \quad (2-14)$$

where  $M$  = the momentum flux transferred across a section per unit width of channel and  $\rho$  = the mass per unit volume.

Substitution of  $dy$  from Equation (2-11) gives

$$\int_0^{u_D} u^2 D f(u) du = M / \rho$$

or

$$\int_0^{u_D} u^2 f(u) du = M / (\rho D) = K_1 \quad (2-15)$$

The equation for conservation of energy in open channel flow is given by

$$D + \left( \int_0^D u^3 dy \right) / \left( 2g \int_0^D u dy \right) = E \quad (2-16)$$

where  $E$  = specific energy flux per unit width of channel at a section, and  $g$  = the acceleration due to gravity.

Substitution of  $dy$  from Equation (2-11) gives

$$D \int_0^{u_D} u^3 f(u) du = (E - D) \left\{ D (2g) \int_0^{u_D} u f(u) du \right\}$$

or

$$\int_0^{u_D} u^3 f(u) du = (E - D) 2g u_m = K_2 u_m \quad (2-17)$$

Therefore the constraints on the system are:

Constraint 1 (from probability)

$$\int_0^{u_D} f(u) \, du = 1 \quad (2-16)$$

where  $u_D$  is the maximum velocity at the water surface.

Constraint 2 (from conservation of mass)

$$\int_0^{u_D} u f(u) \, du = u_m \quad (2-17)$$

where  $u_m$  is the mean velocity (depth-averaged).

Constraint 3 (from conservation of momentum)

$$\int_0^{u_D} u^2 f(u) \, du = K_1 \quad (2-18)$$

where  $K_1 = M / (\rho D)$

Constraint 4 (from conservation of energy)

$$\int_0^{u_D} u^3 f(u) \, du = K_2 u_m \quad (2-19)$$

where  $K_2 = (E - D) / 2 g u_m$

Using the principle of maximum entropy and applying Equations (2-8) and (2-9) we have

$$R(f) = -f(u) \ln f(u) \quad \Rightarrow$$

$$\begin{aligned} \partial R(f) / \partial (f) &= \partial [-f(u) \ln \{f(u)\}] / \partial (f) \\ &= -f(u) * 1/f(u) - \ln \{f(u)\} = -1 - \ln \{f(u)\} \end{aligned}$$

Next we have

$$C_1(f) = f(u) \quad \Rightarrow$$

$$\partial C_1(f) / \partial (f) = \partial f(u) / \partial (f) = 1$$

$$C_2(f) = u f(u) \quad \Rightarrow$$

$$\partial C_2(f) / \partial (f) = \partial \{u f(u)\} / \partial (f) = u \partial f(u) / \partial (f) = u$$

$$C_3(f) = u^2 f(u) \quad \Rightarrow$$

$$\partial C_3(f) / \partial (f) = \partial \{u^2 f(u)\} / \partial (f) = u^2 \partial f(u) / \partial (f) = u^2$$

$$C_4(f) = u^3 f(u) \quad \Rightarrow$$

$$\partial C_4(f) / \partial (f) = \partial \{u^3 f(u)\} / \partial (f) = u^3 \partial f(u) / \partial (f) = u^3$$

The equation to be solved for  $f(u)$  is then

$$-1 - \ln f(u) + L_1 + L_2 u + L_3 u^2 + L_4 u^3 = 0$$

$$\Rightarrow \ln f(u) = L_1 - 1 + L_2 u + L_3 u^2 + L_4 u^3$$

Therefore

$$f(u) = \exp\{L_1 - 1 + L_2 u + L_3 u^2 + L_4 u^3\}$$

Let  $A = L_1 - 1$ , then the probability density function of  $u$  is

$$f(u) = \exp\{A + L_2 u + L_3 u^2 + L_4 u^3\} \quad (2-20)$$

and the velocity distribution is obtained from Equation (2-11) as

$$\int \exp\{A + L_2 u + L_3 u^2 + L_4 u^3\} du = y/D + C \quad (2-21)$$

where  $C$  is the constant of integration to be evaluated at the boundary conditions of  $u = 0$  at  $y = 0$ .

Chiu then let  $L_3 = L_4 = 0$  and solved this equation using only the first two constraints to obtain

$$f(u) = \exp\{L_1 + L_2 u - 1\}$$



or

$$f(u) = \exp(L_1 - 1) \exp(L_2 u) \quad (2-22)$$

where  $L_1$  and  $L_2$  are the Lagrange multipliers. This equation requires only  $u_m$  and  $u_D$  to determine the parameters  $L_1$  and  $L_2$ . Substituting Eq. (2-22) into the first constraint gives

$$\exp(L_1 - 1) = L_2 / \{\exp(L_2 u_D) - 1\} \quad (2-23)$$

Substituting Eq. (2-22) into the second constraint gives

$$u_m = u_D \exp(L_2 u_D) \{\exp(L_2 u_D) - 1\}^{-1} - 1/L_2 \quad (2-24)$$

Substituting the form of the density function, Eq. (2-22), into Eq. (2-11) and integrating gives

$$u = 1/L_2 \ln\{1 + [\exp(L_2 u_D) - 1] y/D\} \quad (2-25)$$

where  $u_D$  and  $L_2$  are the parameters.  $u_D$  and  $L_2$  are related to  $u_m$  by Eq. (2-24).

Equation (2-25) is the entropy based velocity distribution equation, for flow in a wide channel, developed by Chiu. It was developed by using the two constraints of

equations (2-16) and (2-17). Chiu also fit the three and four parameter models based on his constraints to published data. He was therefore able to evaluate the Lagrange coefficients by curve fitting.

In our research, we will solve the three and four constraint models by an approximation technique. We will then compare these approximate solutions to the exact two constraint model solution obtained by Chiu.

## 2.4 Local Scour

### 2.4.1 General

The search for a reliable scour prediction method has been a part of modern hydraulics for about three hundred years. Guglielmini (1697) was one of the first great pioneers in the field of sediment engineering. He was called by Freeman (1929) "the father of the science of river hydraulics". In 1697, he published the first work that dealt with river control and the science of sedimentation. His work was based on field observations.

Frizi (1770) published a work in which he discussed a considerable variety of problems related to river improvements and the transportation of sediment in 1770. He was probably the first to keep records of laboratory studies dealing with sediment transport.

Dubuat (1786) published the first book that was a comprehensive treatment of fluvial hydraulics. In his book he discussed such topics as the formation and migration of sand waves, the stability of channel cross sections, the armoring of river beds, and fluvial morphology. He conducted experiments to determine the velocities necessary to move rock

particles of various sizes.

Baumgarten (1848) published the first measurements of sediment load.

Dupuit (1848) was the first to describe the transportation of sediment in suspension. In 1848 he published his work in which he concluded that the transportation of sediment in suspension was due to the excess velocity on the upper side of the particle as compared with that on the lower side. He also observed that the sediment concentration was greater near the channel bed than near the surface. Further, he noted the effects of various velocities from point to point along the channel.

Du Boys (1879) made the first definitive studies of bed movements in canals and rivers. He presented his theory of "tractive force" in 1879. He stated that the amount of sediment transported by a river was dependent first on the river's slope and then on the river's depth of flow.

In England, Reynolds made his significant contributions to the science of sediment movement in the last half of the nineteenth century. Besides his well known Reynolds number, he developed the techniques of moveable bed model testing.

The process of sediment transportation and deposition was also studied in other sciences besides hydraulic engineering. In the field of geology and geomorphology, Gilbert (1914) studied sediment movement. He conducted extensive laboratory flume studies and published the results in 1914. These studies were conducted at the University of California. This was the most significant data base in the field of sediment movement until the work of Simons and Richardson (1956-1963).

In Germany, Prandtl (1875-1953) was making some of the most important developments of his period. He is considered the founder of modern fluid mechanics. He proposed his concept of the boundary layer in 1901. He helped unite the fields of theoretical hydrodynamics and experimental hydraulics into the modern science of fluid mechanics.

One of the first very complete description of the interaction between flowing water and a mobile alluvial bed was presented by Deacon (1894).

Because of the practical problems encountered in the operation of large irrigation projects in India, much study of the sediment problem was done by Kennedy (1895). He produced the first study that related sediment transport in a channel to the channel shape. He also proposed the

relationship between channel flow velocity and channel flow depth that would produce a stable channel.

Lacey, et al (1929) refined the regime formulas developed by Kennedy (1895) in India for uniform conditions. These formulas have proved to be useful for the steady conditions for which they were developed and are still of use today.

Lane (1955) showed that the shear stress is the appropriate quantity to express critical conditions for sediment movement. He recommended critical shear stress values for the design of irrigation canals.

Inglis (1968) developed a sediment discharge formula by introducing the mean size and fall velocity of the bed sediment and the sediment discharge concentration into the Lacey (1929) regime relation.

Shields (1936) studied plane bed without movement to determine the flow conditions for the beginning of motion. His relation for the incipient motion of sediment, the Shields diagram, is widely accepted (ASCE, 1975). Shields presented a diagram represented by a surface. Rouse (1939) first proposed the line diagram in present use.

Simons and Richardson (1966) derived two friction factor predictors, a slope-adjustment method and a depth-adjustment method, for alluvial channels. They analyzed the extensive data collected by flume experiments at Colorado State University by the United States Geological Survey from 1956 to 1961 (Guy, Simons, and Richardson, 1966).

The concept of unit stream power was used by Yang (1972) and by Yang and Stall (1974) for the determination of sediment transport in natural rivers. He studied in detail the relationship between the rate of energy expenditure and the rate of sediment transport.

#### 2.4.2 Mechanics of Scour

Scour is the net loss of material in a channel's cross section caused by the force of moving water. Since a stream can cause both aggradation and degradation, a more precise definition of scour is when the amount of material removed exceeds the amount deposited.

In the case of a bridge pier, if the rate of transport of sediment away from the region is greater than the rate of transport into the region, a scour hole develops. As the depth of the scour hole increases, the rate of transport of sediment away from the region decreases while the rate of transport into the region stays the same. At some point an equilibrium state exists where the two rates are equal. This is the condition for the maximum scour depth (Laursen, 1952), (Henderson, 1966).

The total scour at any location is made up of the superimposed effects of the three interrelated types of scour, namely, general degradation and aggradation of a river bed, contraction scour, and local scour.

General degradation and aggradation of a river bed is the result of the natural tendency of a river to modify its



channel. This type of process occurs over relatively long reaches and long periods of time. General scour occurs over all parts of the river and may change location within the river's reach with time. This phenomena must be added algebraically to the local and contraction scour to determine the total scour.

Contraction scour is the scour caused by a constriction in the channel's flow. The constriction causes an increase in the velocities in the constricted area. The constriction can be natural or man made. Structures such as bridge piers form an obstruction to the flow which causes a constriction between the piers. Bridges can cause both local and contraction scour.

Local scour is caused by local obstructions or disturbances in the flow. The local scour at a bridge pier is caused by the interference and deflection of the flow around the pier. This interference in the flow causes a bow wave, a pileup of water on the upstream edge of the pier. The subsequent acceleration of the flow around the pier forms the so called "horse shoe" vortex that develops at the pier. The vortex results in high local velocities that erode the bed (Simons and Senturk, 1976).

Local scour can occur in either the live bed or the fixed bed situation. When the drag force caused by the flowing water is not great enough to cause bed motion upstream of the bridge pier, the bed can be considered fixed or rigid. Local scour in this case is called clear water scour.

Live bed scour is local scour that occurs in a bed that is in motion. That is, the bed material upstream of the bridge pier is under sufficient drag force such that particles are passed the point of incipient motion. In this case, there is said to be bed load transport in the river.

Even though the general scour and the contraction scour occur over a large area of the channel bed, their superimposed effects are significant in bridge pier scour. Davoren (1985) found in his field measurements that "local scour was greatest when general scour had occurred; when there had been little or no general scour local scour at the pier was very small". Therefore the greatest bridge scour will occur when the river flow is such as to maximize the algebraic sum of the three types of scour.

Local scour is a function of many variables such as:

- 1) The river bed slope;
- 2) The geometry of the river's cross section;

- 3) The characteristics of the transported sediment;
- 4) The characteristics of the flood hydrograph;
- 5) The antecedent history of recent floods; and
- 6) The geometry and characteristics of the pier's size, shape and composition.

Let us look at the mechanism of local scour at a bridge pier in more detail. When a bridge pier is placed in a river bed, the increase in the local velocity causes erosion of the bed material. This is especially true in channels composed of non-cohesive bed material. When the erosion starts, the flow through the scour hole is in the same general direction as the river flow (FIGURE 1) (Henderson, 1966). The scour hole will be comparatively long and shallow.

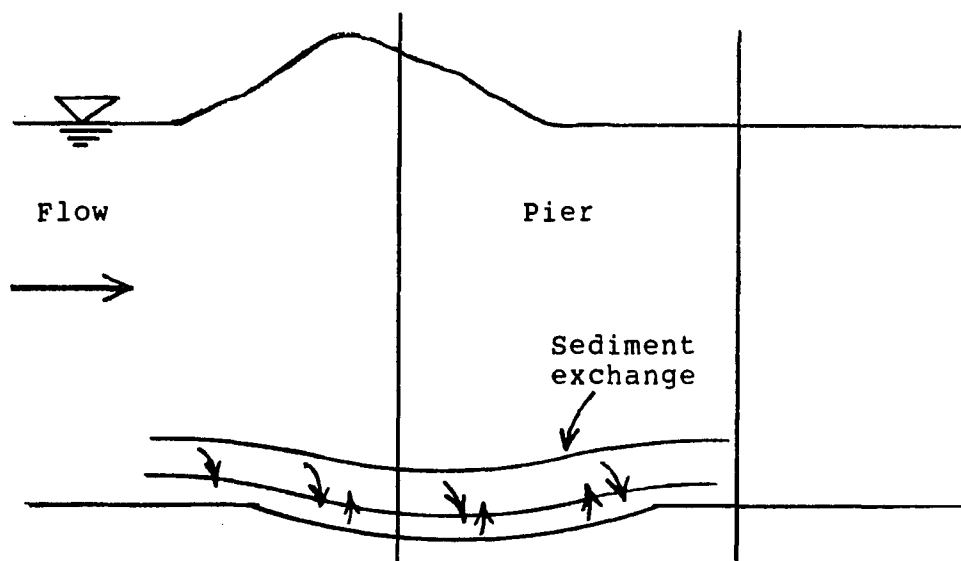


FIGURE 1

As the erosion process continues, the scour hole increases until reverse rollers form (FIGURE 2) (Henderson, 1966). The scour hole will continue to increase until the velocity and shear stress across the base of the scour hole are reduced such that the net erosion over the scour hole is the same as that over the same area of river bed not affected by local scour (Henderson, 1966). In essence, local scour is a balance between material which is continually being picked up from the scour hole into the flow and that which is falling from the flow into the scour hole. The two different particle movements are indicated in FIGURES 1 and 2 by the two sets of arrows showing different directions.

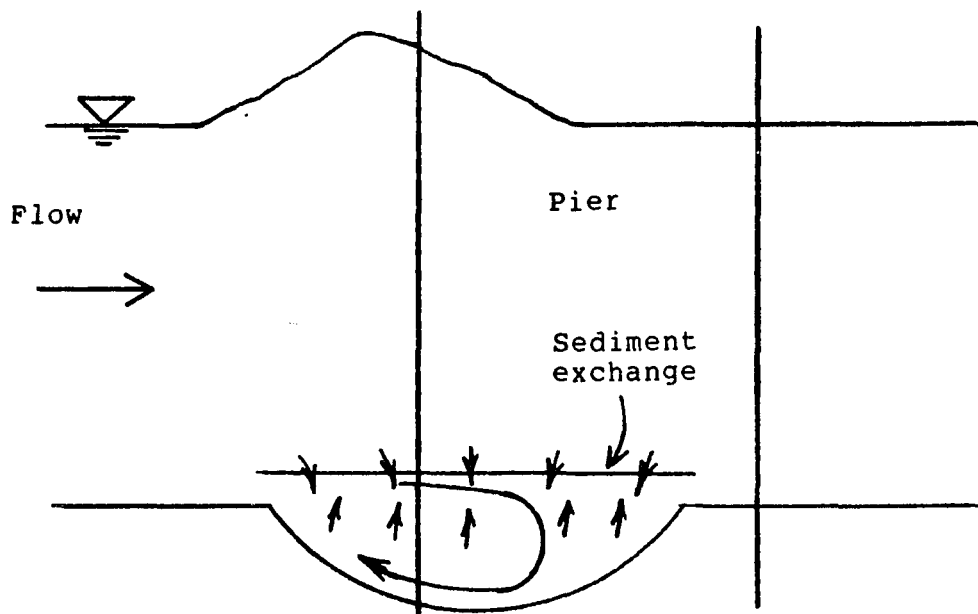


FIGURE 2

Suppose that an equilibrium condition existed where the amount of material being deposited into the scour hole equaled the amount eroded from it. Let us look at the live bed case where an active sediment load exists.

Suppose that the mean size of the bed material was suddenly increased such as to halve the rate of sediment transport upstream of the pier. Also suppose that all velocities and depths were maintained at their previous values. Obviously, while velocities and shear stresses remain unchanged, they are only half as effective in moving the sediment as before because of the increase in size of the particles. Since the velocities and shear stress are only half as effective, the erosion within the scour hole will be half of the previous rate. Therefore the equilibrium between the amount deposited into the scour hole and the amount eroded from it is maintained. This shows that the maximum scour depth is not dependent on the sediment size for the live bed case (Laursen and Toch, 1953; Henderson, 1966).

The previous discussion obviously does not apply to the case of clear water scour. In clear water scour, the bed upstream of the bridge pier is considered fixed or rigid. Therefore there is no deposition of material into the scour hole. The scour hole will increase until the shear stress on

its walls drops to the threshold value for the size of the bed material (Henderson, 1966).

### 2.4.3 Scour Research Methods

The prediction of the maximum depth of scour in any particular situation has depended largely on experimental data from laboratory flumes or experimental channels (ASCE, 1975). This is because as Henderson (1966) stated, "the problem is too complex in its geometry to be theoretically tractable". Any observation of field data that has been available has been used to "verify" the empirically derived formulas.

The method of data collection can have a great impact on the modeling of any natural phenomena. This is especially true of the collection of data from laboratory flumes and experimental channels. Here researchers are collecting information about bridge piers that are only a few centimeters in diameter. Their results of scour depths are then extrapolated for use at actual bridge piers. These experimental set-ups can not conform to all the complexities of the actual field conditions.

The formulas presented here have resulted in a wide range of values for the maximum scour at a bridge pier. This is because they were derived from experiments and then verified with actual data. They were not derived from theoretical analyses.

#### 2.4.4 Scour Prediction Formulas

##### 2.4.4.1 Introduction

There are two basic approaches for the modeling of local scour. The first approach is by regression analysis. This is the type of analysis that has been done by such researchers as Laursen, Lacy, Lane and Kennedy. The second approach is by hydrodynamic analysis. This approach has not been successful as yet. The technique of this research for the study of local scour fits between these two approaches. What will be done is to use both hydrodynamics and regression analysis to better estimate the local scour.

The variables affecting the maximum scour depth at a bridge pier that are commonly used in current scour formulas can be divided into the following groups:

- 1) The geometric characteristics of the bridge pier;
- 2) The characteristics of the flow; and
- 3) The characteristics of the bed material.

The third group is needed in the clear water scour situation.

The first group includes parameters for the pier width and shape. The second group includes parameters for the upstream flow depth and velocity. And the third group



includes parameters for the mean size and gradation of the sediment. None of the equations listed in this section or in Appendix A include all of the factors necessary to describe the previously listed parameters.

Most formulas are obtained by using the parameters that the researcher feels are most important in the scour process. The formula is then derived from one of three methods as follows:

- 1) By the dimensional analysis of these parameters. This involves analysis of geometric, dynamic or kinematic terms.

- 2) By the direct use in the law of conservation of momentum of the parameters believed to be of most importance in the scour process. This involves the use of Reynolds numbers and critical velocities.

- 3) By the direct use in the law of conservation of energy of these parameters. This involves the use of Froude numbers, depths of flow, and the velocity head.

#### 2.4.4.2 Shen, et al's formula

Shen, et al, (1967, 1969) introduced the following formula:

$$d_s = 0.00073 * (Re_{pier})^{0.619} \quad (2-25)$$

where

$d_s$  = the maximum scour depth,

and

$Re_{pier}$  = the pier Reynolds number =  $u_m B / \mu$ ,

$u_m$  = the mean approach velocity,

$B$  = the pier diameter,

$\mu$  = the viscosity.

He derived this formula by using the method of least squares to fit data collected by other researchers. The assumptions used for this formula include subcritical flow and non-cohesive bed material.

His findings were as follows (Strautmann, 1987):

- 1) The bridge pier's shape must be classified as blunt or sharp nose.
- 2) The driving factor for pier scour is the horseshoe vortex in the case of a blunt nosed pier.

- 3) The magnitude of the horseshoe vortex is related to the pier Reynolds number.
- 4) The Froude number of the flow does not provide a good indication of the magnitude of the horseshoe vortex.
- 5) The depth of scour is independent of the sediment material for sands with a size less than 0.52 m .

Shen's popular formulas are a modified form of the previous formulas (Hopkins, 1979):

Shen I

$$d_s/B = 4.43 (F_{\text{pier}})^{2/3} (D/B)^{1/3} \quad (2-26)$$

Shen II

$$d_s/B = 3.4 (F_{\text{pier}})^{2/3} (D/B)^{1/3} \quad (2-27)$$

where

$F_{\text{pier}}$  = the pier Froude number =  $u_m / (g B)^{1/2}$

$u_m$  = the mean approach velocity,

and

$g$  = the acceleration due to gravity.

#### 2.4.4.3 Laursen's formula

Laursen (1962) performed experiments on bridge models with the piers placed in sand. He concluded that the froude number was not an important parameter in bridge scour estimation. He found that the maximum bridge scour depth was related to the upstream flow depth only. His analysis was based on the assumption of a live bed.

His formula can be stated as follows:

$$B/D = 5.5 d_s/D [d_s/(11.5 D) + 1]^{1.7} - 1 \quad (2-28)$$

where

B = the pier diameter,

D = the approach flow depth,

and

$d_s$  = the maximum scour depth.

His findings were as follows (Strautmann, 1987):

- 1) The scour depth was not affected by the proximity of another pier until the scour holes physically overlapped.
- 2) The most important characteristics of the bridge pier were its angle of skew and its ratio of length to width.
- 3) The sediment size does not affect the depth of scour for

the live bed case.

- 4) The approach velocity in the channel does not affect the depth of scour for the live bed case.
- 5) The maximum scour depth was not affected by the degree of contraction caused by the piers until the two scour holes overlapped.

Laursen used experimental flume data to verify his formula.

#### 2.4.4.4 Raudkivi's formula

Raudkivi (1986) collected laboratory flume data using lightweight sediment. He used dimensional analysis and reasoning based on his experiments to develop a graphical relationship for scour. He also assumed subcritical flow.

His findings were as follows (Strautmann, 1987):

- 1) The local scour at a bridge pier is caused by the downflow in front of the pier.
- 2) The scour depth is dependent on the pier width and alignment.
- 3) The scour depth is dependent on the gradation of the sediment.
- 4) The scour depth depends on whether the bed material is ripple forming or not. If it is not ripple forming, then the scour depth =  $2.3 B$ .
- 5) Clear water scour dominates when the shear velocity is less than the critical shear velocity.
- 6) Local scour begins when the shear velocity is greater than half the critical shear velocity. The maximum depth of scour occurs just before the critical shear velocity is reached.
- 7) The scour depth is not dependent on the approach flow depth

when this depth is greater than three pier diameters.

- 8) The greatest values of the local scour occur during unsteady flood flows. This is particularly true when the angle of flow is different at these high flows from that at normal flows.

## 2.5 The horse vortex and vertical velocities

The scour around a bridge pier is caused by a very complex flow pattern around the pier (Figure 3). Tison (1961) described the nature of the flow pattern upstream and around a bridge pier. He showed that a downward flow must exist in front of a bridge pier.

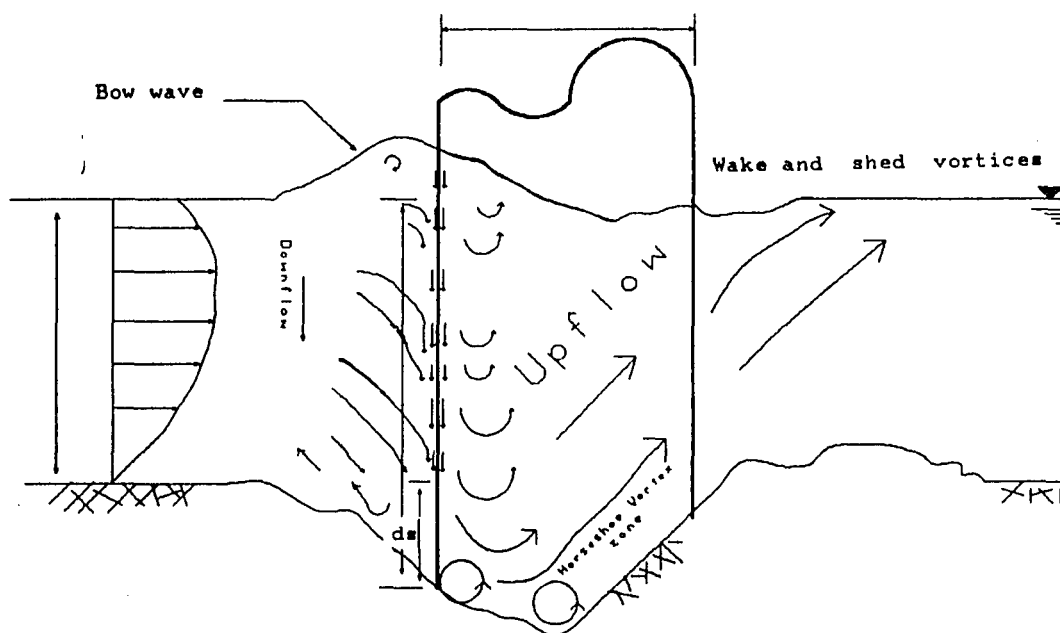


FIGURE 3 (Strautmann, 1987)



This vertical velocity causes the "horseshoe vortex" that is wrapped around the bridge pier near the channel bed. The local scour around the bridge pier is caused by the vortex and the vertical velocity in front of the bridge pier (ASCE, 1975).

As stated by Simons and Senturk (1976), "Although the vortex system is known to be the cause of local scour, it is not possible as yet to calculate the strength of the vortex and relate the velocity field with the subsequent scour".

There is presently no scour formula that utilizes the vertical velocity. This research will derive a scour formula that uses this important feature.

## 2.6 Remarks

Tison (1961) showed that a downward flow exists in front of a bridge pier (Figure 3). This downward flow is the cause of local scour (ASCE, 1975). A bridge pier in a channel does not only cause scour, but also produces a bow wave. The production of the bow wave shows that flow near the surface must be upward. Since it can be expected that the flow closer to the surface does not produce downward vertical flow it should not exert any influence on the scour process. This results in the concept of an effective depth. The effective depth is the portion of the flow which is effective (produces downward vertical flow) in the scour process. This research will develop a method to estimate the effective depth. The horizontal velocity profile within the effective depth will then be related to the maximum depth of scour.

The list of formulas presented shows the interest and the uncertainty of the field of bridge scour estimation. The formulas contain a wide range of assumptions and analytic techniques. They also show a wide range of results. This study is going to compare some of the scour formulas listed here and in Appendix A with the scour method derived in this research. The formulas will be compared to Davoren's (1985) data base.

Readers who are interested in more scour formulas than are described in the previous pages are directed to Appendix A and to the Federal Highway Administration publication (Hopkins, 1979) FHWA-RD-79-103 which list formulas developed before 1969.

## CHAPTER 3

### DESCRIPTION OF DATA BASE

#### 3.1 Introduction

Davoren (1985) made several surveys of the velocity field around a 1.5 m diameter hollow steel cylindrical pier in the Ohau River of New Zealand. He measured the horizontal velocity distribution and the depth of scour as well as the vertical velocity in front of the pier for several flow levels. Davoren's data is the only field measurements where the actual vertical velocity and maximum scour during the peak flow were obtained. The velocities were measured with two Ott propeller current meters that were mounted on a double bracket. The scour depth was measured with a telescopic sounding rod. Measurements were made from January, 1982, through February, 1983.

The site of Davoren's study was downstream of the Ohau-A hydro-electric power station. The discharge from this plant provided flows that were steady for many hours. The discharges ranged from 180 to over 500 m<sup>3</sup>/s. This was equivalent to natural floods with recurrence intervals of up

to 100 years.

The river bed of the Ohau river has a slope of 0.005-0.006 in this area. Also, the bed sediment has a  $d_{50} = 20$  mm.

Davoren (1985) found that the downward component of velocity was greatest in the Ohau River when the approach flow depth was deepest. The downward component of velocity that he measured in the field was generally less than values measured in laboratory flumes (Melville, 1975), (Ettema, 1980). This can account for the fact that the scour depths measured in the field are usually smaller than that predicted by available scour formulas.

Davoren's data represents the only known case of field measurements of both scour and velocities. Therefore this research will use the data base in his study to demonstrate the scour prediction method of this study.

### 3.2 Scour measurements

The maximum depth of scour as measured by Davoren (1985) is presented in Table 1 below. Only his measurements for Runs 1 through 6 were used in the scour part of this study. His data for Runs 7 through 12 were not used because they were taken when the channel was armored.

TABLE 1

| RUN<br>NO. | DATE     | D<br>(m) | $u_m$<br>(m/s) | Q<br>(m <sup>3</sup> /s) | $T_r$<br>(yrs) | $Y_s$<br>(m) |
|------------|----------|----------|----------------|--------------------------|----------------|--------------|
| 1          | 9/20/82  | 3.14     | 2.38           | 480                      | 33.0           | 1.28         |
| 2          | 9/22/82  | 3.02     | 2.69           | 600                      | 100            | 1.20         |
| 3          | 10/06/82 | 2.34     | 2.54           | 360                      | 6.9            | 0.88         |
| 4          | 1/19/82  | 1.44     | 2.65           | 280                      | 2.8            | 0.72         |
| 5          | 1/20/82  | 1.32     | 2.43           | 180                      | 1.2            | 0.59         |
| 6          | 8/09/82  | 1.09     | 2.68           | 300                      | 3.4            | 0.26         |
| 7          | 7/04/82  | 0.94     | 2.39           | 360                      | 6.9            | 0.21         |
| 8          | 7/05/82  | 0.90     | 2.33           | 480                      | 33.0           | 0.20         |
| 9          | 8/11/82  | 0.89     | 2.56           | 360                      | 6.9            | 0.28         |
| 10         | 7/15/82  | 0.70     | 2.24           | 360                      | 6.9            | 0.30         |
| 11         | 4/28/82  | 0.62     | -              | 360                      | 6.9            | 0.39         |
| 12         | 2/21/83  | 0.52     | 2.25           | 360                      | 6.9            | 0.05         |

### 3.3 Horizontal and vertical velocity measurements

The detailed measurements of the horizontal velocity were taken by Davoren (1985) for Runs 1, 3, 6 and 10. Detailed measurements of the vertical velocity were taken for Runs 1 and 10.

Davoren (1985) found that a normal logarithmic profile existed outside of the influence of the pier. This area of influence was about 1.5 m upstream. Within the area of influence of the pier, the horizontal velocity was reduced until it was zero at the face of the pier. Davoren (1985) stated that, "This large reduction in the velocity component parallel to the flow and stream bed was transformed, in part, to both a significant downward component near to and at the nose of the pier, as well as a bow wave at the nose".

The horizontal velocity measurements for Runs 1, 3 and 6 are used in this study. The vertical velocity measurements for Runs 1 and 10 are also used. The horizontal measurements are in Figures 4, 5, and 6. The vertical measurements are in Figures 7 and 8.

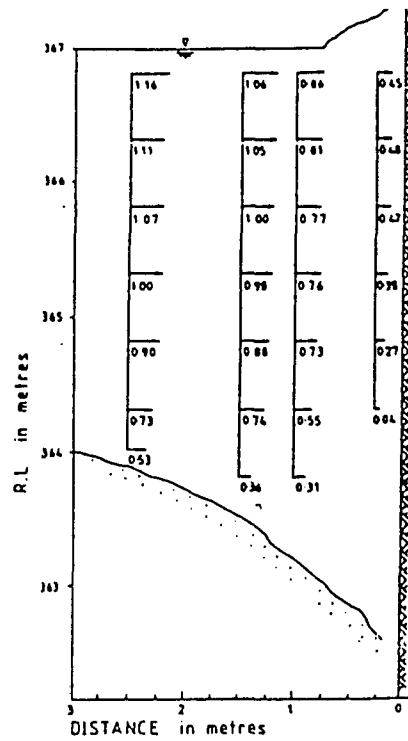


FIGURE 4 Run 1, Davoren (1985)

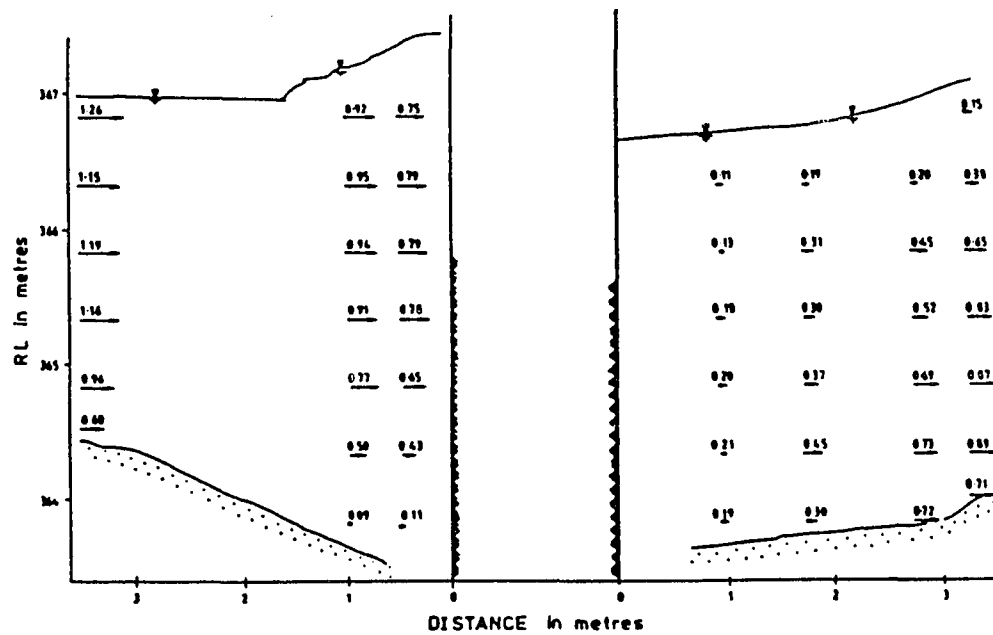


FIGURE 5 Run 3, Davoren (1985)



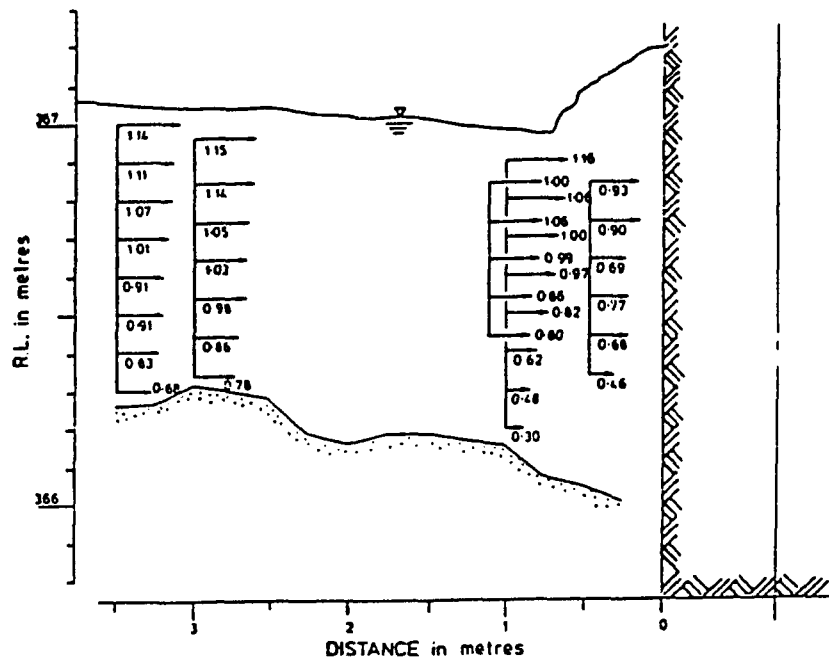


FIGURE 6 Run 6, Davoren (1985)

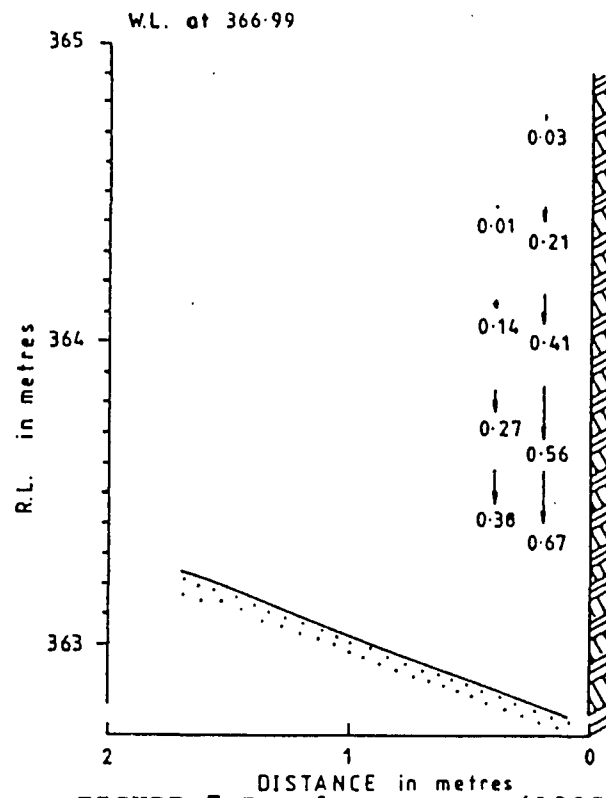


FIGURE 7 Run 1, Davoren (1985)

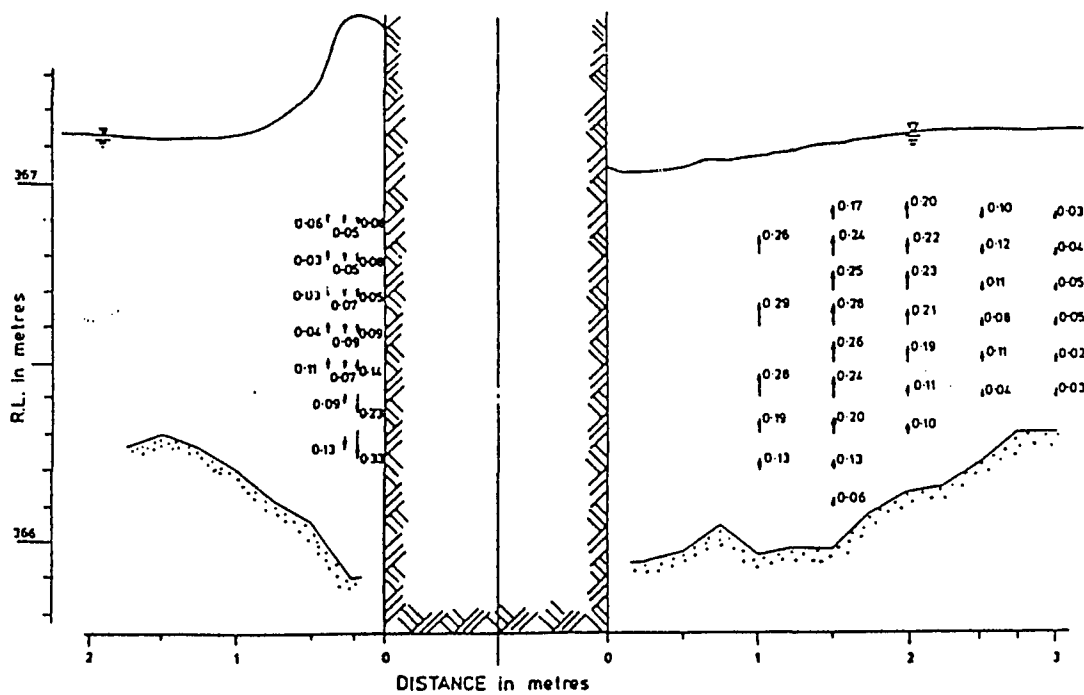


FIGURE 8 Run 10, Davoren (1985)

## CHAPTER 4

### RESEARCH METHODOLOGY

#### 4.1 Introduction

This research will be conducted by performing the following tasks:

1. Determination of the effective depth using the vertical velocities.
2. Derivation of a new horizontal velocity distribution method.
3. Comparison of this method to existing formulas.
4. Relate the horizontal velocity distribution to the scour using the effective depth.
5. Verification of the scour method using real data.

This is the research methodology. In this chapter we will do task 1 through 4.

This research will develop a systematic approach for determining the risk of bridge failure due to scour. This will be accomplished by deriving a scour probability distribution function (PDF) for the bridge pier (chapter 6).

The flood probability distribution function is first computed if it is not available for the site. The scour is then computed for each discharge of the flood PDF. A scour PDF is therefore obtained.

The scour at each discharge will be estimated by means of a new formula developed in this research. This method will use the vertical velocity in front of the pier. There is presently no scour formula that utilizes the vertical velocity.

The vertical velocity will be used as an indication of the strength of the horseshoe vortex. This will be done by defining an effective depth of the horizontal velocity. That is, the amount of the horizontal flow that is deflected downward at the pier. This is the part of the horizontal flow that causes the scour at the pier. The horizontal velocity profile within the effective depth will be used to derive a scour equation.

The horizontal velocity distribution is an important parameter in this research's determination of the maximum scour. The theory of entropy is used to develop a method for computing the horizontal velocity distribution more accurately. The procedure produces a system of equations that

are not exactly solvable by analytical means. A solution by approximation is presented. The velocity distribution obtained by the entropy method is compared to other methods by means of historical data.

The method outlined above is then applied to a pier and the results compared to the field data in chapter 6.

#### 4.2 Definition of the horse-shoe vortex from the effective depth of the horizontal velocity

The flow around a bridge pier is shown in Figure 9 below. Some of the approaching flow is deflected upward by the pier as evidenced by the formation of a bow wave. Also, some of the flow is deflected downward as evidenced by the formation of a scour hole. The scour is caused by the downward flow in front of the pier which forms a "horseshoe" vortex at the base of the pier.

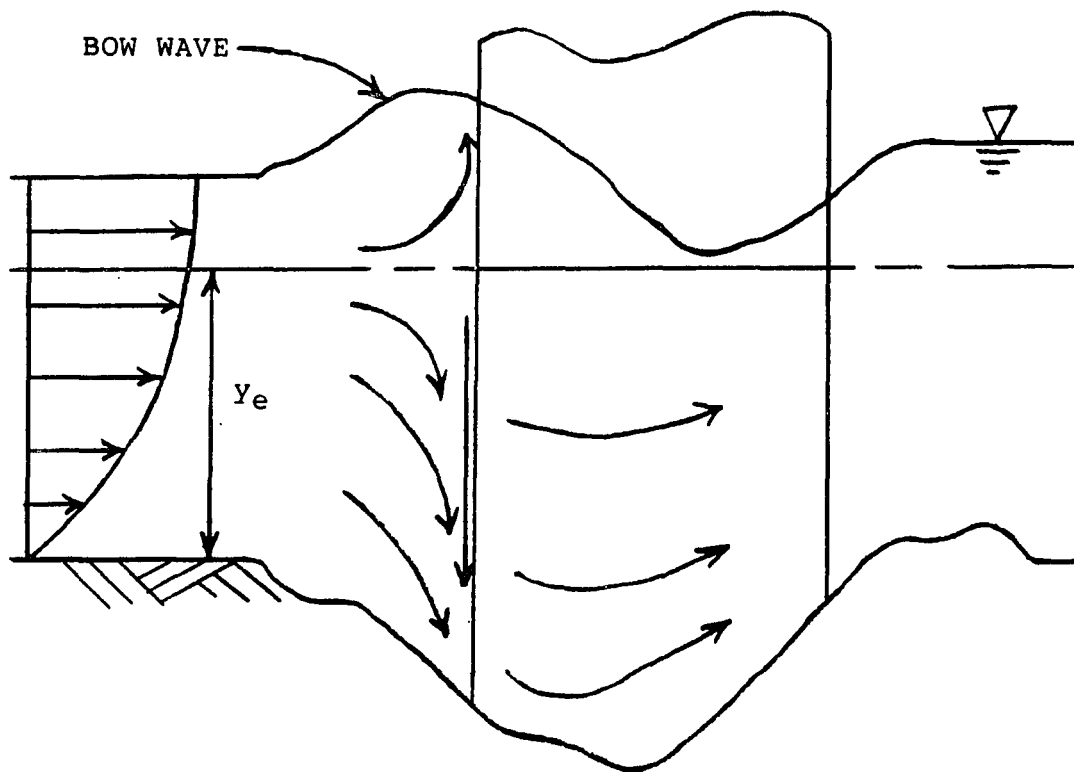


FIGURE 9

This research will define the effective depth as that part of the approach flow that is deflected downward, that is, the part of the flow that is effective in causing scour. This depth is labeled  $y_e$  in Figure 9.

Davoren (1985) measured downflow velocities at the nose of a pier in the field. He found that the downflow velocities increased closer to the pier and closer to the scour hole. His measurements of these velocities were limited by the measurement technique. The Ott current meters and the bracket holding them did not allow him to measure downflow velocities extremely close to the pier (less than 0.2 m).

The downflow velocity measurements used in this study are the ones Davoren (1985) observed about 0.2 m from the pier. The point where downflow became significant was taken as the depth where the vertical component of the velocity exceeded 0.03 of the mean approach velocity which was the smallest measurement observable. The effective depth of the horizontal velocity was taken as the distance from the point of significant vertical velocity to the bed elevation of the approach flow.

The observed data will be used to obtain a correlation between the approach depth and the effective depth. The

effective depth will then be obtainable for any approach depth that is determined from the flood PDF.

The effective depth was determined for the two complete sets of downflow observations of Davoren (Run 1 and 10). These two effective depth data points were plotted with respect to the actual approach depth (Figure 10). A curve was then passed through these points and the point of zero effective depth, which occurs at an approach depth of zero.

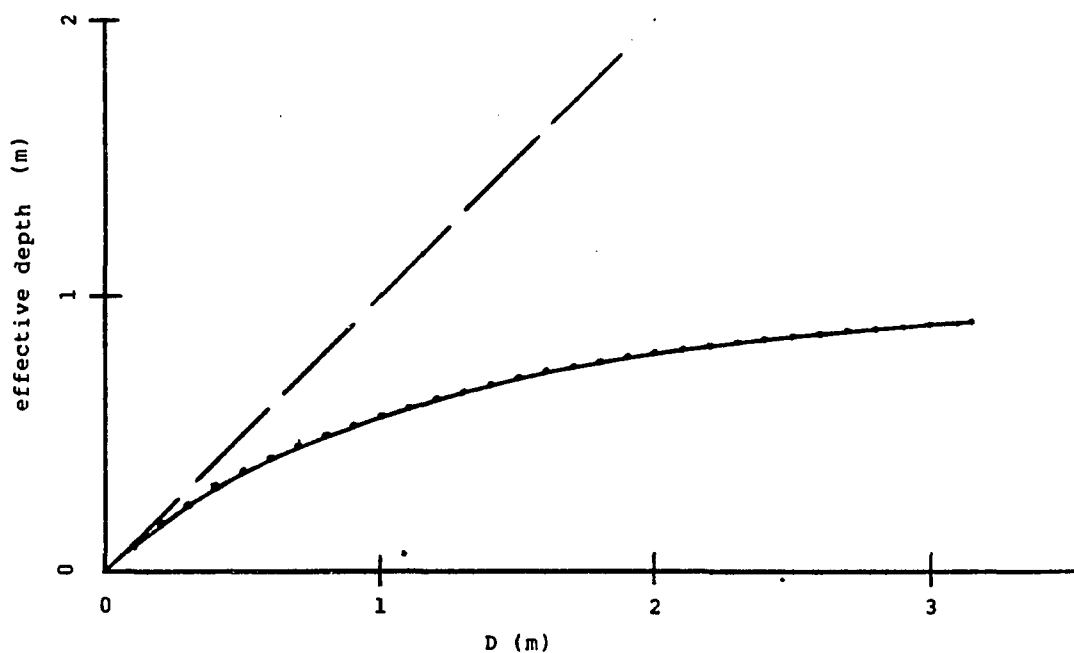


FIGURE 10



To fit the physical situation (so that the curve used does not exceed the  $y_e = y$  line), the derivative of the function was made equal to the derivative of  $y_e = y$  at  $D = 0$ . That is the boundary condition at  $D = 0$  was "clamped" such that the first derivative is 1. The boundary condition at  $D = 3.14$  was "unclamped", that is left free. This means that the boundary condition at  $D = 3.14$  is the second derivative is 0. The polynomial was obtained by the method of cubic spline (Burden and Faires, 1985).

The equation produced for the effective depth was

$$y_e = D - 0.6945 D^2 + 0.2633 D^3 \quad (0 \leq D \leq 0.7 \text{ m}) \quad (4-89)$$

and

$$y_e = 0.45 + 0.4147(D - .7) - 0.1416(D - .7)^2 + 0.0193(D - .7)^3 \quad (0.7 \text{ m} \leq D \leq 3.14 \text{ m}) \quad (4-90)$$

where

$y_e$  = the effective depth of horizontal flow (in meters)

and

$D$  = the total approach flow depth (in meters).

### 4.3 Derivation of the velocity distribution using POME

The entropy principle states that in a steady equilibrium condition a system tends to maximize the entropy under prevailing constraints.

Let  $u$  be the velocity at a vertical distance  $y$  from the channel bed. Let  $u_D$  be the maximum velocity which occurs at the surface. From Chiu (1987, 1988) we have the constraints of:

Constraint 1:

$$\int_0^{u_D} f(u) \, du = 1 \quad (4-1)$$

Constraint 2:

$$\int_0^{u_D} u f(u) \, du = u_m \quad (4-2)$$

where  $u_m$  = the mean velocity (depth averaged).

Constraint 3

$$\int_0^{u_D} u^2 f(u) \, du = M / (\rho D) = K_1 \quad (4-5)$$

Constraint 4

$$\int_0^{u_D} u^3 f(u) du = (E - D) / 2 \cdot g \cdot u_m = K_2 \cdot u_m \quad (4-6)$$

Applying the principle of maximum entropy, the probability density function is given as (Chiu, 1989)

$$f(u) = \exp\{L_1 - 1 + L_2 u + L_3 u^2 + L_4 u^3\} \quad (4-7)$$

Let  $A = L_1 - 1$ , then

$$f(u) = \exp\{A + L_2 u + L_3 u^2 + L_4 u^3\} \quad (4-8)$$

Substituting  $f(u)$  into constraint 1 gives

$$\int_0^{u_D} \exp\{A + L_2 u + L_3 u^2 + L_4 u^3\} du = 1 \quad (4-9)$$

Substituting  $f(u)$  into constraint 2 gives

$$\int_0^{u_D} u \exp\{A + L_2 u + L_3 u^2 + L_4 u^3\} du = u_m \quad (4-10)$$

Substituting  $f(u)$  into constraint 3 gives

$$\int_0^{u_D} u^2 \exp\{A + L_2 u + L_3 u^2 + L_4 u^3\} du = K_1 \quad (4-11)$$

Substituting  $f(u)$  into constraint 4 gives

$$\int_0^{u_D} u^3 \exp\{A + L_2 u + L_3 u^2 + L_4 u^3\} du = K_2 u_m \quad (4-12)$$

The constraint equations, Equations (4-9), (4-10), (4-11), and (4-12), must be solved simultaneously for the unknown Lagrange coefficients  $A$ ,  $L_2$ ,  $L_3$ , and  $L_4$ . These integral equations do not have exact analytical solutions. Their solution will be obtained by approximation of the exponential function within the integral.

After we have a solution for the Lagrange coefficients, we obtain the velocity distribution as follows:

From differentiation of the CDF we have

$$f(u) = 1/D \, dy/du$$

Substitution of  $f(u)$  gives

$$\exp\{A + L_2 u + L_3 u^2 + L_4 u^3\} = 1/D \, dy/du$$

or

$$\int \exp\{A + L_2 u + L_3 u^2 + L_4 u^3\} du = \int 1/D \, dy$$

Therefore

$$\int \exp\{A + L_2 u + L_3 u^2 + L_4 u^3\} du = y/D + C \quad (4-13)$$

The boundary conditions for the evaluation of C are  $u = 0$  at  $y = 0$ .

#### 4.4 Approximation of the entropy distribution

The entropy method gives the velocity pdf in Equ. (4-8), where  $A$ ,  $L_2$ ,  $L_3$ , and  $L_4$  are the Lagrange coefficients obtained from solving the constraint equations of Equ. (4-9) to (4-12).

If we let  $L_3 = L_4 = 0$ , we obtain the entropy based, two constraint, velocity distribution derived by Chiu (1987).

If we let  $L_4 = 0$ , we obtain the entropy based, three constraint, velocity distribution based on momentum as follows:

$$f(u) = \exp \{ A + L_2 u + L_3 u^2 \} \quad (4-14)$$

where the constraints are:

Constraint 1

$$\int_0^{u_D} \exp\{A + L_2 u + L_3 u^2\} du = 1 \quad (4-15)$$

Constraint 2

$$\int_0^{u_D} u \exp\{A + L_2 u + L_3 u^2\} du = u_m \quad (4-16)$$

Constraint 3

$$\int_0^{u_D} u^2 \exp\{A + L_2 u + L_3 u^2\} du = K_1 \quad (4-17)$$

These equations are not solvable by exact analytic means. An approximate solution is obtained by expansion of the term involving the third parameter,  $L_3$ .

For a function  $f(x) = \exp(g(x))$  the Maclaurin Series expansion is given by

$$f(x) = 1 + [g(x)] + [g(x)]^2/2! + [g(x)]^3/3! + \dots$$

For  $f(u) = \exp(L_3 u^2)$  we have

$$f(u) = 1 + L_3 u^2 + (L_3 u^2)^2/2! + (L_3 u^2)^3/3! + \dots$$

Using the first two terms of the Maclaurin Series expansion, the constraint equations become:

Constraint 1

$$\int_0^{u_D} \exp\{A + L_2 u\} * \{1 + L_3 u^2\} du = 1 \quad (4-18)$$

Constraint 2

$$\int_0^{u_D} u \exp\{A + L_2 u\} * \{1 + L_3 u^2\} du = u_m \quad (4-19)$$

Constraint 3

$$\int_0^{u_D} u^2 \exp\{A + L_2 u\} * \{1 + L_3 u^2\} du = K_1 \quad (4-20)$$

The velocity distribution becomes

$$\int \exp\{A + L_2 u\} * \{1 + L_3 u^2\} du = y/D + C \quad (4-21)$$

where  $u = 0$  at  $y = 0$ .

Solving the constraints by integration by parts gives:

From Constraint 1

$$\begin{aligned} L_2 \exp(-A) &= \{\exp(L_2 u_D) - 1\} \\ + L_3 \{\exp(L_2 u_D) [u_D^2 - 2u_D/L_2 + 2/L_2^2] - 2/L_2^2\} \end{aligned} \quad (4-22)$$

From Constraint 2

$$\begin{aligned} L_2 u_m \exp(-A) &= \{\exp(L_2 u_D) [u_D - 1/L_2] + 1/L_2\} \\ + L_3 \{\exp(L_2 u_D) [u_D^3 - 3u_D^2/L_2 + 6u_D/L_2^2 - 6/L_2^3] - 6/L_2^3\} \end{aligned} \quad (4-23)$$



From Constraint 3

$$\begin{aligned}
 L_2 K_1 \exp(-A) = & \{ \exp(L_2 u_D) [u_D^2 - 2u_D/L_2 + 2/L_2^2] - 2/L_2^2 \} \\
 + L_3 \{ & \exp(L_2 u_D) [u_D^4 - 4u_D^3/L_2 + 12u_D^2/L_2^2 - 24u_D/L_2^3 + 24/L_2^4] \\
 - & 24/L_2^4 \}
 \end{aligned}
 \tag{4-24}$$

Solving these equations simultaneously we obtain

$$\begin{aligned}
 & \{ \exp(L_2 u_D) [u_D - 1/L_2 - u_m] + 1/L_2 + u_m \} * \\
 & \{ \exp(L_2 u_D) [K_1 u_D^2 - 2u_D K_1/L_2 + 2K_1/L_2^2 - u_D^4 + 4u_D^3/L_2 \\
 & - 12u_D^2/L_2^2 + 24u_D/L_2^3 - 24/L_2^4] - 2K_1/L_2^2 + 24/L_2^4 \} \\
 = & \{ \exp(L_2 u_D) [u_D^2 - 2u_D/L_2 + 2/L_2^2 - K_1] - 2/L_2^2 + K_1 \} * \\
 & \{ \exp(L_2 u_D) [u_m u_D^2 - 2u_m u_D/L_2 + 2u_m/L_2^2 - u_D^3 + 3u_D^2/L_2 \\
 & - 6u_D/L_2^2 + 6/L_2^3] - 2u_m/L_2^2 + 6/L_2^3 \}
 \end{aligned}
 \tag{4-25}$$

where  $K_1 = M / (\rho D)$ . This equation is solved by iteration for  $L_2$ .

The value of  $L_3$  is obtained by substitution of  $L_2$  into the equation

$$\begin{aligned}
 L_3 = & \{ \exp(L_2 u_D) [u_D - 1/L_2 - u_m] + 1/L_2 + u_m \} / \\
 & \{ \exp(L_2 u_D) [u_m u_D^2 - 2u_m u_D/L_2 + 2u_m/L_2^2 - u_D^3 + 3u_D^2/L_2 \\
 & - 6u_D/L_2^2 + 6/L_2^3] - 2u_m/L_2^2 + 6/L_2^3 \}
 \end{aligned}
 \tag{4-26}$$

The coefficient A is then obtained by substitution of  $L_2$  and  $L_3$  into equation (4-22).

The velocity distribution is solved by the Maclaurin Series expansion to give

$$\begin{aligned} & \exp(A) \{ \exp(L_2 u_D) + L_3 [\exp(L_2 u_D) (u_D^2 - 2u_D/L_2 + 2/L_2^2)] \} \\ = & y/D + C \end{aligned} \quad (4-27)$$

At  $y = 0$  ,  $u = 0$  therefore

$$C = \exp(A) \{ 1/L_2 + 2L_3/L_2^3 \} \quad (4-28)$$

The velocity distribution equation for the two term Maclaurin Series expansion of the three constraint entropy method based on momentum is

$$\begin{aligned} & \exp(A) \{ \exp(L_2 u_D) + L_3 [\exp(L_2 u_D) (u_D^2 - 2u_D/L_2 + 2/L_2^2)] \} \\ = & y/D + \exp(A) \{ 1/L_2 + 2L_3/L_2^3 \} \end{aligned} \quad (4-29)$$

If we let  $L_3 = 0$  in the four constraint entropy method, we obtain the three constraint entropy method based on energy as follows:

$$f(u) = \exp \{A + L_2 u + L_4 u^3\} \quad (4-30)$$

The constraints are:

Constraint 1

$$\int_0^{u_D} \exp\{A + L_2 u + L_4 u^3\} du = 1 \quad (4-31)$$

Constraint 2

$$\int_0^{u_D} u \exp\{A + L_2 u + L_4 u^3\} du = u_m \quad (4-32)$$

Constraint 3

$$\int_0^{u_D} u^2 \exp\{A + L_2 u + L_4 u^3\} du = K_2 u_m \quad (4-33)$$

The velocity distribution is therefore

$$\int \exp\{A + L_2 u + L_4 u^3\} du = y/D + C \quad (4-34)$$

where  $u = 0$  at  $y = 0$ .

These equations can be approximated by using the Maclaurin Series expansion of the term involving the fourth parameter,  $L_4$ .

The Maclaurin Series for  $f(u) = \exp(L_4 u^3)$  is given by

$$f(u) = 1 + L_4 u^3 + (L_4 u^3)^2/2! + (L_4 u^3)^3/3! + \dots \quad (4-35)$$

If we use the first two terms to approximate the function, the constraint equations become:

Constraint 1

$$\int_0^{u_D} \exp\{A + L_2 u\} * \{1 + L_4 u^3\} du = 1 \quad (4-36)$$

Constraint 2

$$\int_0^{u_D} u \exp\{A + L_2 u\} * \{1 + L_4 u^3\} du = u_m \quad (4-37)$$

Constraint 3

$$\int_0^{u_D} u^2 \exp\{A + L_2 u\} * \{1 + L_4 u^3\} du = K_2 u_m \quad (4-38)$$

The velocity distribution becomes

$$\int \exp\{A + L_2 u\} * \{1 + L_4 u^3\} du = y/D + C \quad (4-39)$$

Solving the constraint equations by integration by parts gives:

From Constraint 1

$$\begin{aligned} L_2 \exp(-A) &= \{\exp(L_2 u_D) - 1\} \\ &+ L_4 \{\exp(L_2 u_D) [u_D^3 - 3u_D^2/L_2 + 6u_D/L_2^2 - 6/L_2^3] + 6/L_2^3\} \end{aligned} \quad (4-40)$$

From Constraint 2

$$\begin{aligned} L_2 u_m \exp(-A) &= \{\exp(L_2 u_D) [u_D - 1/L_2] + 1/L_2\} \\ &+ L_4 \{\exp(L_2 u_D) [u_D^4 - 4u_D^3/L_2 + 12u_D^2/L_2^2 - 24/L_2^3 + 24/L_2^4] \\ &- 24/L_2^4\} \end{aligned} \quad (4-41)$$

From Constraint 3

$$\begin{aligned} L_2 K_2 u_m \exp(-A) &= \\ &\{\exp(L_2 u_D) [u_D^3 - 3u_D^2/L_2 + 6u_D/L_2^2 - 6/L_2^3] + 6/L_2^3\} \\ &+ L_4 \{\exp(L_2 u_D) [u_D^6 - 6u_D^5/L_2 + 30u_D^4/L_2^2 - 120u_D^3/L_2^3 \\ &+ 360u_D^2/L_2^4 - 720u_D/L_2^5 + 720/L_2^6] - 720/L_2^6\} \end{aligned} \quad (4-42)$$

These three equations are solved simultaneously to obtain

$$\begin{aligned}
 & \{ \exp(L_2 u_D) [u_D - 1/L_2 - u_m] + 1/L_2 + u_m \} * \\
 & \{ \exp(L_2 u_D) [K_2 u_m u_D^3 - 3u_D^2 K_2 u_m / L_2 + 6K_2 u_m u_D / L_2^2 - 6K_2 u_m / L_2^3 \\
 & - u_D^6 + 6u_D^5 / L_2 - 30u_D^4 / L_2^2 + 120u_D^3 / L_2^3 - 360u_D^2 / L_2^4 \\
 & + 720u_D / L_2^5 - 720 / L_2^6] + 6K_2 u_m / L_2^3 - 720 / L_2^6 \} \\
 & = \{ \exp(L_2 u_D) [u_D^3 - 3u_D^2 / L_2 + 6u_D / L_2^2 - 6 / L_2^3 - K_2 u_m] + K_2 u_m \} \\
 & * \{ \exp(L_2 u_D) [u_m u_D^3 - 3u_m u_D^2 / L_2 + 6u_m u_D / L_2^2 - 2u_m / L_2^3 - u_D^4 \\
 & + 4u_D^3 / L_2 - 12u_D^2 / L_2^2 + 24u_D / L_2^3 - 24 / L_2^4] + 6u_m / L_2^3 - 24 / L_2^4 \} \\
 & \quad (4-43)
 \end{aligned}$$

where  $K_2 = 2g(E - D)$ . This equation can be solved for  $L_2$  by iteration.

The value of  $L_4$  is obtained by substitution of  $L_2$  into the equation

$$\begin{aligned}
 L_4 = & \{ \exp(L_2 u_D) [u_D - 1/L_2 - u_m] + 1/L_2 + u_m \} / \\
 & \{ \exp(L_2 u_D) [u_D^3 - 3u_D^2 / L_2 + 6u_D / L_2^2 - 6 / L_2^3 - K_2 u_m] + K_2 u_m \} \\
 & * \{ \exp(L_2 u_D) [u_m u_D^3 - 3u_m u_D^2 / L_2 + 6u_m u_D / L_2^2 - 2u_m / L_2^3 - u_D^4 \\
 & + 4u_D^3 / L_2 - 12u_D^2 / L_2^2 + 24u_D / L_2^3 - 24 / L_2^4] + 6u_m / L_2^3 - 24 / L_2^4 \} \\
 & \quad (4-44)
 \end{aligned}$$

The value of  $A$  is obtained by substitution of  $L_2$  and  $L_4$  into equation (4-40).

The velocity distribution then becomes

$$\begin{aligned} \exp(A) \{ \exp(L_2 u) / L_2 \\ + L_4 [ \exp(L_2 u) / L_2 (u^3 - 3u^2 / L_2 + 6u / L_2^2 - 6 / L_2^3) ] \} = y/D + C \end{aligned} \quad (4-45)$$

At  $y = 0$ ,  $u = 0$  therefore

$$C = \exp(A) \{ 1/L_2 - 6L_4/L_2^4 \} \quad (4-46)$$

The velocity distribution for the two term Maclaurin Series expansion for the three constraint entropy method based on energy is

$$\begin{aligned} y/D + \exp(A) \{ 1/L_2 - 6L_4/L_2^4 \} \\ = \exp(A) \{ \exp(L_2 u) / L_2 \\ + L_4 [ \exp(L_2 u) / L_2 (u^3 - 3u^2 / L_2 + 6u / L_2^2 - 6 / L_2^3) ] \} \end{aligned} \quad (4-47)$$

Using all four constraints in the entropy method, we have

$$f(u) = \exp \{A + L_2 u + L_3 u^2 + L_4 u^3\} \quad (4-8)$$

where the constraints are as follows:

Constraint 1

$$\int_0^{u_D} \exp\{A + L_2 u + L_3 u^2 + L_4 u^3\} du = 1 \quad (4-48)$$

Constraint 2

$$\int_0^{u_D} u \exp\{A + L_2 u + L_3 u^2 + L_4 u^3\} du = u_m \quad (4-49)$$

Constraint 3

$$\int_0^{u_D} u^2 \exp\{A + L_2 u + L_3 u^2 + L_4 u^3\} du = K_1 \quad (4-50)$$

Constraint 4

$$\int_0^{u_D} u^3 \exp\{A + L_2 u + L_3 u^2 + L_4 u^3\} du = K_2 u_m \quad (4-51)$$



The velocity distribution is therefore

$$\int \exp\{A + L_2 u + L_3 u^2 + L_4 u^3\} du = y/D + C \quad (4-52)$$

where  $u = 0$  at  $y = 0$ .

The Maclaurin Series expansion for  $\exp(L_3 u^2 + L_4 u^3)$  is

$$\begin{aligned} \exp(L_3 u^2 + L_4 u^3) = 1 + (L_3 u^2 + L_4 u^3) + (L_3 u^2 + L_4 u^3)^2/2! \\ + (L_3 u^2 + L_4 u^3)^3/3! + \dots \end{aligned} \quad (4-53)$$

If we use the first two terms of the Maclaurin Series expansion we obtain the following:

Constraint 1

$$\int_0^{u_D} \exp\{A + L_2 u\} * \{1 + L_3 u^2 + L_4 u^3\} du = 1 \quad (4-54)$$

Constraint 2

$$\int_0^{u_D} u \exp\{A + L_2 u\} * \{1 + L_3 u^2 + L_4 u^3\} du = u_m \quad (4-55)$$

Constraint 3

$$\int_0^{u_D} u^2 \exp\{A + L_2 u\} * \{1 + L_3 u^2 + L_4 u^3\} du = K_1 \quad (4-56)$$

Constraint 4

$$\int_0^{u_D} u^3 \exp\{A + L_2 u\} * \{1 + L_3 u^2 + L_4 u^3\} du = K_2 u_m \quad (4-57)$$

The velocity distribution becomes

$$\int \exp\{A + L_2 u\} * \{1 + L_3 u^2 + L_4 u^3\} du = y/D + C \quad (4-58)$$

Solving the constraint equations by integration by parts the following equations are obtained:

From Constraint 1

$$\begin{aligned} L_2 \exp(-A) &= \{\exp(L_2 u_D) - 1\} \\ &+ L_3 \{\exp(L_2 u_D) [u_D^2 - 2u_D/L_2 + 2/L_2^2] - 2/L_2^2\} \\ &+ L_4 \{\exp(L_2 u_D) [u_D^3 - 3u_D^2/L_2 + 6u_D/L_2^2 - 6/L_2^3] + 6/L_2^3\} \end{aligned} \quad (4-59)$$

From Constraint 2

$$\begin{aligned}
 L_2 u_m \exp(-A) &= \{ \exp(L_2 u_D) [u_D - 1/L_2] + 1/L_2 \} \\
 &+ L_3 \{ \exp(L_2 u_D) [u_D^3 - 3u_D^2/L_2 + 6u_D/L_2^2 - 6/L_2^3] - 6/L_2^3 \} \quad + \\
 &L_4 \{ \exp(L_2 u_D) [u_D^4 - 4u_D^3/L_2 + 12u_D^2/L_2^2 - 24/L_2^3 + 24/L_2^4] \\
 &- 24/L_2^4 \}
 \end{aligned}
 \tag{4-60}$$

From Constraint 3

$$\begin{aligned}
 L_2 K_1 \exp(-A) &= \{ \exp(L_2 u_D) [u_D^2 - 2u_D/L_2 + 2/L_2^2] - 2/L_2^2 \} \\
 &+ L_3 \{ \exp(L_2 u_D) [u_D^4 - 4u_D^3/L_2 + 12u_D^2/L_2^2 - 24u_D/L_2^3 + 24/L_2^4] \\
 &- 24/L_2^4 \} \\
 &+ L_4 \{ \exp(L_2 u_D) [u_D^5 - 5u_D^4/L_2 + 20u_D^3/L_2^2 - 60u_D^2/L_2^3 \\
 &+ 120u_D/L_2^4 - 120/L_2^5] + 120/L_2^5 \}
 \end{aligned}
 \tag{4-61}$$

From Constraint 4

$$\begin{aligned}
 L_2 K_2 u_m \exp(-A) &= \\
 &\{ \exp(L_2 u_D) [u_D^3 - 3u_D^2/L_2 + 6u_D/L_2^2 - 6/L_2^3] + 6/L_2^3 \} \\
 &+ L_3 \{ \exp(L_2 u_D) [u_D^5 - 5u_D^4/L_2 + 20u_D^3/L_2^2 - 60u_D^2/L_2^3 \\
 &+ 120u_D/L_2^4 - 120/L_2^5] + 120/L_2^5 \} \\
 &+ L_4 \{ \exp(L_2 u_D) [u_D^6 - 6u_D^5/L_2 + 30u_D^4/L_2^2 - 120u_D^3/L_2^3 \\
 &+ 360u_D^2/L_2^4 - 720u_D/L_2^5 + 720/L_2^6] - 720/L_2^6 \}
 \end{aligned}
 \tag{4-62}$$

Solving these equations simultaneously gives

$$\begin{aligned}
 & [(u_m \alpha_1 - \alpha_2)(K_1 \beta_1 - \beta_3) - (K_1 \alpha_1 - \alpha_3)(u_m \beta_1 - \beta_2)] * \\
 & [(\gamma_3 - K_1 \gamma_1)(K_2 u_m \beta_1 - \beta_4) + (K_2 u_m \gamma_1 - \gamma_4)(K_1 \beta_1 - \beta_3)] \\
 & = [(\gamma_2 - u_m \gamma_1)(K_1 \beta_1 - \beta_3) + (K_1 \gamma_1 - \gamma_3)(u_m \beta_1 - \beta_2)] * \\
 & [(K_1 \alpha_1 - \alpha_3)(K_2 u_m \beta_1 - \beta_4) - (K_2 u_m \alpha_1 - \alpha_4)(K_1 \beta_1 - \beta_3)]
 \end{aligned}
 \tag{4-63}$$

where

$$\alpha_1 = \{\exp(L_2 u_D) - 1\} \tag{4-64}$$

$$\beta_1 = \{\exp(L_2 u_D) [u_D^2 - 2u_D/L_2 + 2/L_2^2] - 2/L_2^2\} \tag{4-65}$$

$$\gamma_1 = \{\exp(L_2 u_D) [u_D^3 - 3u_D^2/L_2 + 6u_D/L_2^2 - 6/L_2^3] + 6/L_2^3\} \tag{4-66}$$

$$\alpha_2 = \{\exp(L_2 u_D) [u_D - 1/L_2] + 1/L_2\} \tag{4-67}$$

$$\beta_2 = \{\exp(L_2 u_D) [u_D^3 - 3u_D^2/L_2 + 6u_D/L_2^2 - 6/L_2^3] - 6/L_2^3\} \tag{4-68}$$

$$\begin{aligned}
 \gamma_2 = \{ & \exp(L_2 u_D) [u_D^4 - 4u_D^3/L_2 + 12u_D^2/L_2^2 - 24u_D/L_2^3 + 24/L_2^4] \\
 & - 24/L_2^4 \}
 \end{aligned}
 \tag{4-69}$$

$$\alpha_3 = \{\exp(L_2 u_D) [u_D^2 - 2u_D/L_2 + 2/L_2^2] - 2/L_2^2\} \quad (4-70)$$

$$\begin{aligned} \beta_3 = \{\exp(L_2 u_D) [u_D^4 - 4u_D^3/L_2 + 12u_D^2/L_2^2 - 24u_D/L_2^3 + 24/L_2^4] \\ - 24/L_2^4\} \end{aligned} \quad (4-71)$$

$$\begin{aligned} \gamma_3 = \{\exp(L_2 u_D) [u_D^5 - 5u_D^4/L_2 + 20u_D^3/L_2^2 - 60u_D^2/L_2^3 \\ + 120u_D/L_2^4 - 120/L_2^5] + 120/L_2^5\} \end{aligned} \quad (4-72)$$

$$\alpha_4 = \{\exp(L_2 u_D) [u_D^3 - 3u_D^2/L_2 + 6u_D/L_2^2 - 6/L_2^3] + 6/L_2^3\} \quad (4-73)$$

$$\begin{aligned} \beta_4 = \{\exp(L_2 u_D) [u_D^5 - 5u_D^4/L_2 + 20u_D^3/L_2^2 - 60u_D^2/L_2^3 \\ + 120u_D/L_2^4 - 120/L_2^5] + 120/L_2^5\} \end{aligned} \quad (4-74)$$

$$\begin{aligned} \gamma_4 = \{\exp(L_2 u_D) [u_D^6 - 6u_D^5/L_2 + 30u_D^4/L_2^2 - 120u_D^3/L_2^3 \\ + 360u_D^2/L_2^4 - 720u_D/L_2^5 + 720/L_2^6] - 720/L_2^6\} \end{aligned} \quad (4-75)$$

Equation (4-63) can be solved for  $L_2$  by iteration.  $L_4$  is then obtained by substitution of  $L_2$  into the following equation:

$$L_4 = [ (u_m \alpha_1 - \alpha_2) (\kappa_1 \beta_1 - \beta_3) - (\kappa_1 \alpha_1 - \alpha_3) (u_m \beta_1 - \beta_2) ] / [ (\gamma_2 - u_m \gamma_1) (\kappa_1 \beta_1 - \beta_3) + (\kappa_1 \gamma_1 - \gamma_3) (u_m \beta_1 - \beta_2) ] \quad (4-76)$$

$L_3$  is obtained by substitution of  $L_2$  and  $L_4$  into the following equation:

$$L_3 = [-L_4 (u_m \gamma_1 - \gamma_2) - (u_m \alpha_1 - \alpha_2)] / (u_m \beta_1 - \beta_2) \quad (4-77)$$

The value of  $A$  is obtained by substitution of  $L_2$ ,  $L_3$ , and  $L_4$  into equation (4-59).

Solving the velocity distribution equation gives

$$\begin{aligned} & \exp(-A) \{ \{ \exp(L_2 u) / L_2 + L_3 \{ \exp(L_2 u) / L_2 (u^2 - 2u/L_2 + 2/L_2^2) \} \\ & + L_4 \{ \exp(L_2 u) / L_2 (u^3 - 3u^2/L_2 + 6u/L_2^2 - 6/L_2^3) \} \} \\ & = y/D + C \end{aligned} \quad (4-78)$$

where  $u = 0$  at  $y = 0$ .

Therefore

$$C = \exp(A) [1/L_2 + 2L_3/L_2^3 - 6L_4/L_2^4] \quad (4-79)$$

The velocity distribution for the four constraint entropy method with the Maclaurin Series approximation is

$$\begin{aligned} & \exp(-A) \left[ \frac{\exp(L_2 u)}{L_2} + L_3 \left\{ \frac{\exp(L_2 u)}{L_2} (u^2 - 2u/L_2 + 2/L_2^2) \right\} \right. \\ & \left. + L_4 \left\{ \frac{\exp(L_2 u)}{L_2} (u^3 - 3u^2/L_2 + 6u/L_2^2 - 6/L_2^3) \right\} \right] \\ & = y/D + \exp(A) [1/L_2 + 2L_3/L_2^3 - 6L_4/L_2^4] \end{aligned} \quad (4-80)$$

Both of the three parameter formulas will be used in the following evaluations. To use them, the assumption that  $\alpha = \beta = 1$  will be used. Therefore both the momentum and energy can be obtained using the mean velocity  $u_m$ .

The theory of entropy states that the entropy of a system is a measure of the information content of the data. The above assumption does not add any new information when both the momentum and energy constraints are used. The data necessary to use both the momentum and energy together does not presently exist. Therefore the four parameter formula will be reserved for future research.

#### 4.5 Relative error of the Maclaurin expansions

In this section we will look at the Taylor series remainder term for each of the Maclaurin series expansions of the three parameter entropy method. This will give us an idea as to which method would be expected to give the least error. The actual error of the method is not computed, just the bounds for the error associated with the approximation by series truncation.

For the three constraint entropy method based on momentum, the actual Maclaurin Series expansion of  $\exp(L_3 u^2)$  is given by

$$\exp(L_3 u^2) = 1 + L_3 u^2 + R_n(u) \quad (4-81)$$

The remainder term  $R_n(u)$  is given by

$$|R_n(u)| = |[8L_3^3 z^3 + 12L_3^2 z] \exp(L_3 z^2) u^3/6| \quad (4-82)$$

where  $z$  is some number between 0 and  $u$ .

Since  $0 \leq u \leq u_D$  the remainder term is bounded by

$$|R_n(u)| \leq |[8L_3^3 u_D^3 + 12L_3^2 u_D] \exp(L_3 u_D^2) u^3/6| \quad (4-83)$$

Therefore the maximum error is at  $u = u_D$ ,

$$|R_n(u_D)| \leq |[8L_3^3 u_D^6 + 12L_3^2 u_D^4] \exp(L_3 u_D^2)/6| \quad (4-84)$$



For the three constraint entropy method based on energy, the actual Maclaurin Series expansion of  $\exp(L_4 u^3)$  is given by

$$\exp(L_4 u^3) = 1 + L_4 u^3 + R_n(u) \quad (4-85)$$

The remainder term  $R_n(u)$  is given by

$$R_n(u) = [81L_4^4 z^7 + 135L_4^3 z^4 + 162L_4^3 z^5 + 180L_4^2 z^2] \exp(L_4 z^3) u^4/24 \quad (4-86)$$

where  $z$  is some number between 0 and  $u$ .

Since  $0 \leq u \leq u_D$  the remainder term is bounded by

$$|R_n(u)| \leq |[81L_4^4 u_D^7 + 135L_4^3 u_D^4 + 162L_4^3 u_D^5 + 180L_4^2 u_D^2] * \exp(L_4 u_D^3) u^4/24| \quad (4-87)$$

Again the maximum error is at  $u = u_D$ ,

$$|R_n(u)| \leq |[81L_4^4 u_D^{11} + 135L_4^3 u_D^8 + 162L_4^3 u_D^9 + 180L_4^2 u_D^6] * \exp(L_4 u_D^3)| \quad (4-88)$$

The error for the energy constraint Eq. (4-88) has  $u_D$  to the eleventh power while the error for the momentum constraint Eq. (4-84) has  $u_D$  to the sixth power. For natural channels that we will consider,  $u_D > 1$ . Therefore if  $L_3$  and  $L_4$  are of similar magnitudes, we can expect that the error in the three

parameter momentum method will be less than the three  
parameter energy method.

#### 4.6 Derivation of a Scour Equation

Most scour prediction formulas of the past have used average horizontal velocity and depth of flow. This is because even though it has been known that the horse shoe vortex system around a pier and its related downward flow is the cause of local scour, no method for the computation of the strength of the vortex existed.

This study will use the effective depth of horizontal velocity as a measure of this strength and relate it to the local scour. Since the downflow is significant only within the effective depth, it seems reasonable that this portion of the approach flow is most significant in causing scour.

This approach seems reasonable in view of the findings of Davoren (1985) and others. He observed that upstream beyond the influence of the pier, the horizontal velocity conformed to a logarithmic profile. But closer to the pier, the horizontal component of velocity was slowed. This component of flow parallel to the bed was transformed into a significant downward component. It is this component of velocity that causes scour.

As shown by Strautmann(1987) and others, the specific

energy is a good indicator of the scour. The specific energy is measured in units of length ( foot-pounds per pound or Newton-meters per Newton). Therefore an equation for scour that is based on the specific energy would be dimensionally correct. This study will therefore examine the specific energy within the effective depth of the horizontal flow.

The equation to be investigated will have the form

$$d_s = C_1 y_e + \left\{ C_2 \int_0^{y_e} u^3 dy \right\} / \left\{ 2 g \int_0^{y_e} u dy \right\} \quad (4-91)$$

where

$d_s$  = the maximum depth of local scour,

$y_e$  = the effective depth of vertical velocity at the pier,

$u$  = the horizontal velocity at a depth  $y$  above the bed,

$g$  = the acceleration due to gravity,

and

$C_1$  and  $C_2$  = constants to be determined (Energy loss coefficients).

#### 4.7 Remarks

The method outlined in this study uses measurements of the vertical velocity to estimate the effective depth of horizontal flow. The method proposed in this study will therefore require either the field measurement of the vertical velocity or flume studies of bridge models to approximate the effective depth for each site evaluated. This data is not easily obtainable at a bridge site that is being evaluated. Therefore, flume studies of different pier shapes, material, and width/depth ratios to determine a relationship between the approach flow and the effective depth would greatly facilitate the use of this scour estimation method. The effective depth obtained is site specific.

Many researchers, such as Laursen and Toch (1953), have stated that the maximum scour is dependent on the approach flow depth only. These method usually give results that are higher than actually measured in the field. The coefficient  $C_1$  can therefore be thought of as the indicator of the maximum scour while  $C_2$  is a correction factor. Again, as stated for the effective depth, the scour equation is site specific. That is, the values of  $C_1$  and  $C_2$  will be different for different pier shapes, material, and width/depth ratios. We can not do a sensitivity analysis for these characteristics

but only for changing discharge.

This research will study local scour only in the live bed case. For the clear water case, it can be expected that a relationship between the approach flow and the effective depth would be required for each different sediment size. The procedure for the estimation of the scour process would than be similar to that for live bed except that the equilibrium condition would change. Equilibrium would occur when the scour process is equal to resistance forces related to the sediment characteristics.

## CHAPTER 5 - RESULTS

### 5.1 Comparison of velocity distribution methods

#### 5.1.1 Introduction

There are three basic methods to obtain a time averaged horizontal velocity distribution that will be evaluated. They are the logarithmic distribution law of Prandtl - von Karman, the power law, and the entropy method.

This research will compare the two parameter entropy distribution derived by Chiu (1987) as well as the three parameter momentum based entropy distributions derived in this work. The four parameter entropy distribution derived in this work will be reserved for future study.

All the distributions will be computed for the variables presented in the work of Davoren (1985). This is actual field data for a pier in a river with a live-bed. Each theoretical distribution is plotted and compared to the actual velocity distribution measured in the field. A comparison to Run 1 will be shown in this chapter and the remaining comparisons to Runs 3, 6, and 10 are shown in the Appendix C.

### 5.1.2 Prandtl and von Karman

The Prandtl - von Karman universal logarithmic velocity distribution law can be stated as follows:

$$u = u_D + (g D S)^{1/2} / K \ln(y / D) \quad (2-1)$$

where D = the depth of flow in the channel,

u = the horizontal velocity at a distance y from the channel bed,

K = the von Karman universal constant which has a value of 0.40 for clear water and a value as low as 0.2 in flows with heavy sediment loads,

$u_D$  = the maximum velocity of the flow,

g = the acceleration due to gravity,

and

S = the channel slope.

The velocity distribution obtained from this equation for the data of Run 1 is tabulated in Table 2 and plotted against the observed profile in Figure 11. Only the profile near the bed above a depth of 0.25 m is tabulated in Table 2, the entire distribution is given in Appendix B.



| DEPTH (m) | VELOCITY (m/s)                |      |      |
|-----------|-------------------------------|------|------|
|           | von Karman universal constant |      |      |
|           | 0.2                           | 0.3  | 0.4  |
| 0.25      | -                             | -    | 0.20 |
| 0.30      | -                             | -    | 0.38 |
| 0.35      | -                             | -    | 0.54 |
| 0.40      | -                             | -    | 0.68 |
| 0.45      | -                             | 0.13 | 0.80 |
| 0.50      | -                             | 0.28 | 0.91 |
| 0.55      | -                             | 0.41 | 1.01 |
| 0.60      | -                             | 0.53 | 1.10 |
| 0.65      | -                             | 0.64 | 1.18 |
| 0.70      | -                             | 0.74 | 1.26 |
| 0.75      | -                             | 0.84 | 1.33 |
| 0.80      | -                             | 0.92 | 1.39 |
| 0.85      | 0.11                          | 1.01 | 1.46 |
| 0.90      | 0.23                          | 1.09 | 1.51 |
| 0.95      | 0.34                          | 1.16 | 1.57 |
| 1.00      | 0.45                          | 1.23 | 1.62 |
| 1.05      | 0.55                          | 1.30 | 1.67 |
| 1.10      | 0.64                          | 1.36 | 1.72 |
| 1.15      | 0.73                          | 1.42 | 1.77 |
| 1.20      | 0.82                          | 1.48 | 1.18 |
| 1.25      | 0.90                          | 1.54 | 1.85 |

Table 2

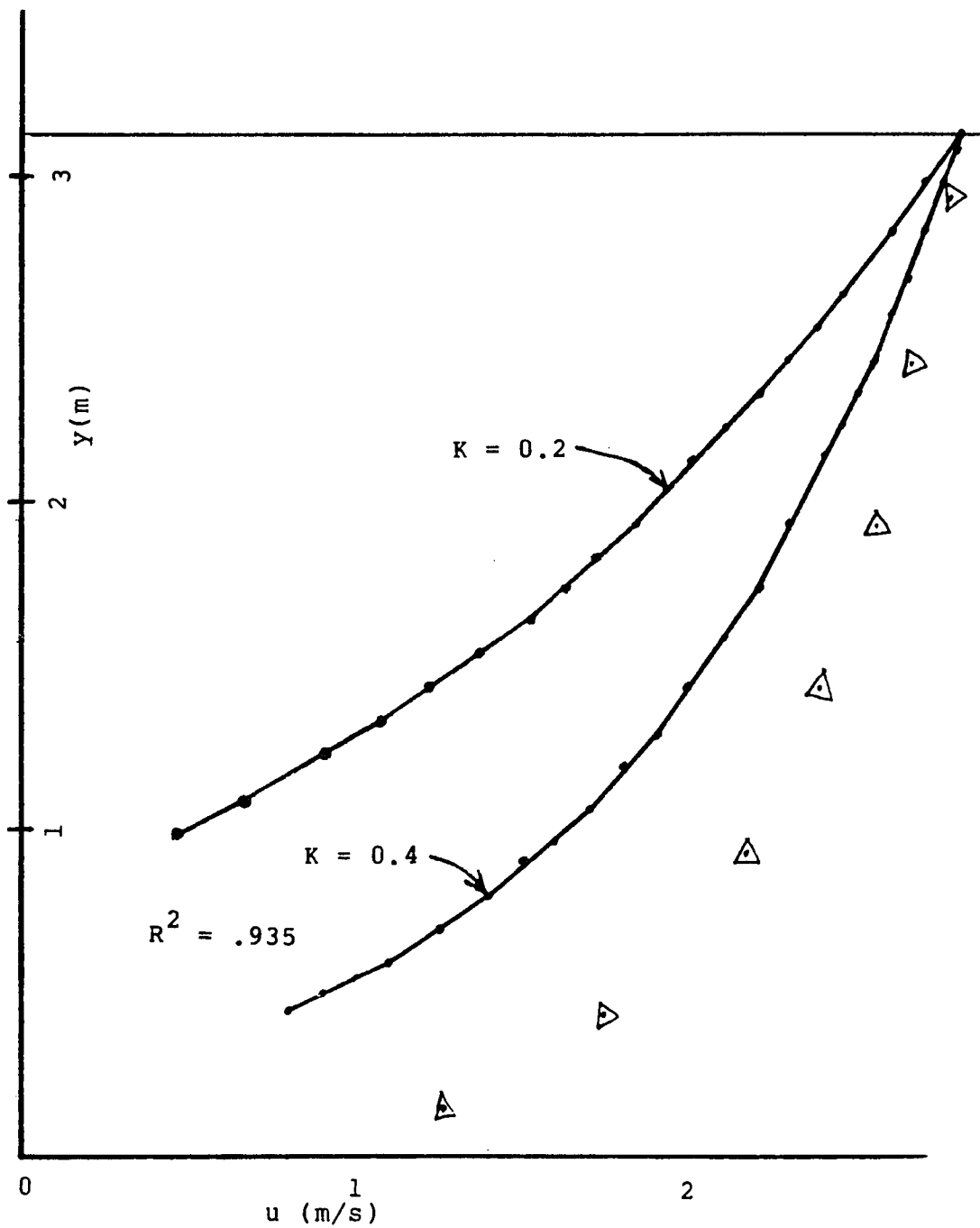


Figure 11

### 5.1.3 Simple power law

The power law velocity distribution for flow in an open channel can be stated as follows:

$$u / u_D = (y / D)^{(1/n)} \quad (2-4)$$

where

$u$  = the velocity at a vertical depth of  $y$  above the channel bed,

$u_D$  = the maximum velocity,

$D$  = the total depth of the flow,

and

$n$  = a parameter determined by the frictional resistance at the bed ( $n$  is usually in the range of 6-7).

The velocity distribution obtained from this equation for Run 1 of the data is tabulated in Table 3 and plotted against the observed profile in Figure 12. Only the profile near the bed is tabulated in Table 3, the entire distribution is given in Appendix B.

| DEPTH (m) | VELOCITY (m/s) |      |
|-----------|----------------|------|
|           | coefficient n  |      |
|           | 6              | 7    |
| 0.05      | 1.40           | 1.55 |
| 0.10      | 1.58           | 1.71 |
| 0.15      | 1.69           | 1.81 |
| 0.20      | 1.80           | 1.89 |
| 0.25      | 1.84           | 1.95 |
| 0.30      | 1.89           | 2.00 |
| 0.35      | 1.94           | 2.05 |
| 0.40      | 1.99           | 2.09 |
| 0.45      | 2.03           | 2.12 |
| 0.50      | 2.06           | 2.15 |
| 0.55      | 2.09           | 2.18 |
| 0.60      | 2.13           | 2.21 |
| 0.65      | 2.15           | 2.24 |
| 0.70      | 2.18           | 2.26 |
| 0.75      | 2.21           | 2.28 |
| 0.80      | 2.23           | 2.30 |
| 0.85      | 2.25           | 2.32 |
| 0.90      | 2.27           | 2.34 |
| 0.95      | 2.29           | 2.36 |
| 1.00      | 2.31           | 2.38 |
| 1.05      | 2.33           | 2.39 |

Table 3

| DEPTH (m) | VELOCITY (m/s) |      |
|-----------|----------------|------|
|           | coefficient n  |      |
|           | 6              | 7    |
| 0.05      | 1.40           | 1.55 |
| 0.10      | 1.58           | 1.71 |
| 0.15      | 1.69           | 1.81 |
| 0.20      | 1.80           | 1.89 |
| 0.25      | 1.84           | 1.95 |
| 0.30      | 1.89           | 2.00 |
| 0.35      | 1.94           | 2.05 |
| 0.40      | 1.99           | 2.09 |
| 0.45      | 2.03           | 2.12 |
| 0.50      | 2.06           | 2.15 |
| 0.55      | 2.09           | 2.18 |
| 0.60      | 2.13           | 2.21 |
| 0.65      | 2.15           | 2.24 |
| 0.70      | 2.18           | 2.26 |
| 0.75      | 2.21           | 2.28 |
| 0.80      | 2.23           | 2.30 |
| 0.85      | 2.25           | 2.32 |
| 0.90      | 2.27           | 2.34 |
| 0.95      | 2.29           | 2.36 |
| 1.00      | 2.31           | 2.38 |
| 1.05      | 2.33           | 2.39 |

Table 3

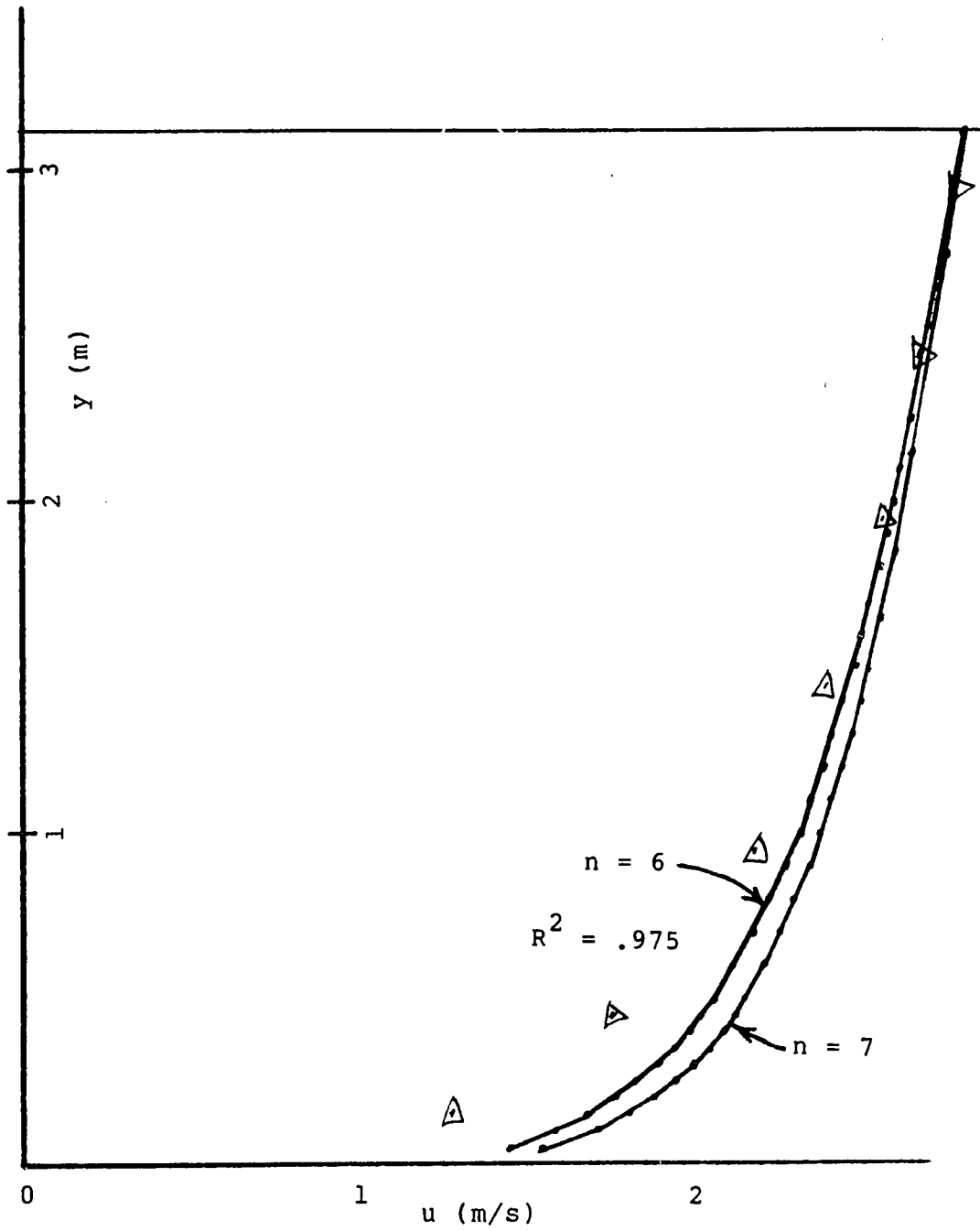


Figure 12

#### 5.1.4 Entropy method

The two parameter entropy velocity distribution is given by

$$u = 1/L_2 \ln[(L_2 y)/(\exp\{A\} D) + 1] \quad (2-24)$$

where

$$\exp(A) = L_2/[\exp(L_2 u_D) - 1] \quad (2-22)$$

and

$$u_m = \{u_D \exp(L_2 u_D)\} / \{\exp(L_2 u_D) - 1\} - 1/L_2 \quad (2-23)$$

Given  $u_m$  and  $u_D$ , this can be solved for  $L_2$ . The previous equation is then solved for  $A$ .

To use the three constraint momentum method, the assumption that  $\beta = 1$  was used. Therefore Constraint 3 becomes:

$$\int_0^{u_D} u^2 f(u) du = u_m^2 \quad (4-5)$$

and then  $K_1 = u_m^2$ . The only parameters are then  $u_m$  and  $u_D$ .

The three parameter momentum based entropy velocity distribution is given by

$$\begin{aligned} & \exp(A) \{ \exp(L_2 u_D) + L_3 [\exp(L_2 u_D) (u_D^2 - 2u_D/L_2 + 2/L_2^2)] \} \\ = & y/D + \exp(A) \{ 1/L_2 + 2L_3/L_2^3 \} \end{aligned} \quad (4-29)$$

where  $L_2$  is found from Equ. (4-25) by iteration with  $K_1 = u_m^2$ . The value of  $L_3$  and  $A$  are obtained from Equ. (4-26) and Equ. (4-22) respectively.

To use the three constraint energy method, the assumption that  $\alpha = 1$  was used. Therefore Constraint 4 becomes:

$$\int_0^{u_D} u^3 f(u) du = u_m^3 \quad (4-5)$$

and then  $K_2 = u_m^2$ . The only parameters are then  $u_m$  and  $u_D$ . The three parameter energy based entropy velocity distribution is given by

$$\begin{aligned} & \exp(A) \{ \exp(L_2 u)/L_2 \\ & + L_4 [\exp(L_2 u)/L_2 (u^3 - 3u^2/L_2 + 6u/L_2^2 - 6/L_2^3)] \} = y/D + C \end{aligned}$$

where  $L_2$  is found from Equation (4-43) by iteration with  $K_2 = u_m^2$ . The value of  $L_4$  and  $A$  are obtained from Equ. (4-44)

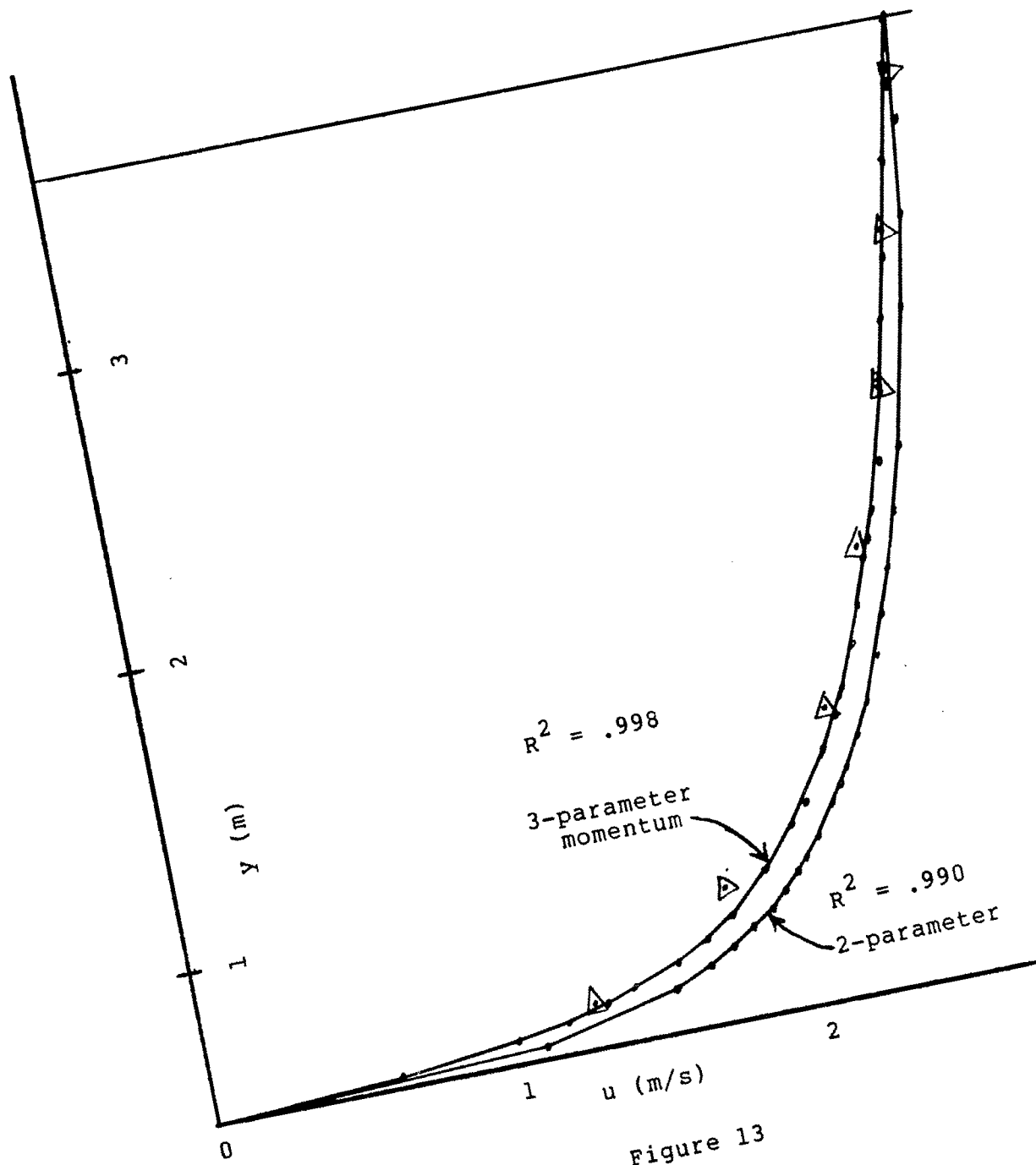


and Equation (4-40) respectively.

The velocity distributions obtained from the entropy methods for Run 1 of the data are tabulated in Table 4 and plotted against the observed profile in Figure 13. Only the profile near the bed is tabulated in Table 4, the entire distribution is given in Appendix B.

| DEPTH (m) | VELOCITY (m/s) |            |            |
|-----------|----------------|------------|------------|
|           | EQN (2-24)     | EQN (4-29) | EQN (4-47) |
| 0.05      | 1.08           | 0.91       | 1.17       |
| 0.10      | 1.36           | 1.20       | 1.42       |
| 0.15      | 1.52           | 1.38       | 1.57       |
| 0.20      | 1.64           | 1.51       | 1.68       |
| 0.25      | 1.73           | 1.61       | 1.76       |
| 0.30      | 1.80           | 1.69       | 1.83       |
| 0.35      | 1.87           | 1.76       | 1.89       |
| 0.40      | 1.93           | 1.83       | 1.94       |
| 0.45      | 1.98           | 1.88       | 1.99       |
| 0.50      | 2.02           | 1.93       | 2.03       |
| 0.55      | 2.06           | 1.97       | 2.07       |
| 0.60      | 2.10           | 2.01       | 2.10       |
| 0.65      | 2.13           | 2.05       | 2.13       |
| 0.70      | 2.17           | 2.09       | 2.16       |
| 0.75      | 2.19           | 2.12       | 2.19       |
| 0.80      | 2.22           | 2.15       | 2.22       |
| 0.85      | 2.25           | 2.18       | 2.24       |
| 0.90      | 2.27           | 2.20       | 2.26       |
| 0.95      | 2.29           | 2.23       | 2.29       |
| 1.00      | 2.32           | 2.25       | 2.31       |
| 1.05      | 2.34           | 2.27       | 2.33       |
| 1.10      | 2.36           | 2.30       | 2.35       |

Table 4



### 5.1.5 Remarks and Conclusions

The fit of the Prandtl - von Karman logarithmic velocity distribution was only good close to the surface of the flow. As the value of  $y$  is decreased, the departure from the observed data of the equation becomes apparent ( $R^2 = .935$ ). The logarithmic velocity distribution was totally unacceptable near the bed of the channel. Therefore it seems reasonable to conclude that this velocity distribution would not be appropriate for any evaluation of near bed processes such as local scour.

The velocity distribution for the power law was good for a value of  $n = 6$  ( $R^2 = .975$ ). Again, this distribution fit the observed data best near the channel surface and departed from this fit as the value of  $y$  is decreased. Even though the fit of the power law velocity distribution was better than that of the logarithmic velocity distribution near the channel bed, we still would like more accuracy for use in the determination of near bed processes.

The two parameter entropy velocity distribution had a superior fit to the observed data than the two previously mentioned velocity distributions ( $R^2 = .990$ ). The fit of this velocity distribution was not only good at the surface of the

channel but also was good near the channel bed.

The velocity distributions obtained by the three parameter momentum based entropy method had the best fit to the observed data of any of the methods compared ( $R^2 = .998$ ). The momentum based velocity distribution had a better fit than that of the energy based method (not shown). This was because of the approximation technique.

The Maclaurin series expansion of the  $L_3$  term used in the momentum based method converges more rapidly than the  $L_4$  term used in the energy method. This is as stated in the error analysis and because although both  $L_3$  and  $L_4$  are small, the value of  $L_3$  is one order of magnitude smaller than the value of  $L_4$ .

The fit of the three parameter momentum based entropy velocity distribution was particularly good near the bed of the channel (Run 1 was shown in this chapter and Runs 3, 6, and 10 are in Appendix C). This implies that this velocity distribution should be good for the description of near bed processes such as local scour. This velocity distribution will be used in this research.

## 5.2 Evaluation of scour estimation methods

### 5.2.1 Introduction

The scour will be computed by several popular methods for the data in Davoren's (1985) research. The scour will be computed for his stated Runs 1, 3, and 6. The information required in the scour formulas is tabulated below in Table 5 where B is the pier width.

| RUN                         | 1     | 3     | 6     |
|-----------------------------|-------|-------|-------|
| $u_m$ (m/s)                 | 2.38  | 2.54  | 2.68  |
| D (m)                       | 3.14  | 2.34  | 1.09  |
| $y_s$ (m)                   | 1.28  | 0.88  | 0.26  |
| B (m)                       | 1.5   | 1.5   | 1.5   |
| $\mu$ (N s/m <sup>2</sup> ) | 0.001 | 0.001 | 0.001 |

TABLE 5

### 5.2.2 Shen, et al's formula

Shen's formula (Shen, et al, 1967) can be stated as follows:

Shen II

$$d_s/B = 3.4 (F_{\text{pier}})^{2/3} (D/B)^{1/3} \quad (2-27)$$

where

$D$  = the approach flow depth,

$B$  = the pier diameter,

$F_{\text{pier}}$  = the pier Froude number =  $u_m / (g B)^{1/2}$

$u_m$  = the mean approach velocity,

and

$g$  = the acceleration due to gravity.

RUN 1

$$d_s = 1.5 [ 3.4 (.620)^{2/3} (3.14/1.5)^{1/3} ] = 4.75 \text{ m}$$

RUN 3

$$d_s = 1.5 [ 3.4 (.662)^{2/3} (2.34/1.5)^{1/3} ] = 4.49 \text{ m}$$

RUN 6

$$d_s = 1.5 [ 3.4 (.699)^{2/3} (1.09/1.5)^{1/3} ] = 3.61 \text{ m}$$

### 5.2.3 Laursen's formula

Laursen's formula (Laursen, 1962) can be stated as follows:

$$B/D = 5.5 d_s/D [d_s/(11.5 D) + 1]^{1.7} - 1 \quad (2-28)$$

where

B = the pier diameter,

D = the approach flow depth,

and

$d_s$  = the maximum scour depth.

RUN 1

$$1.5/3.14 = 5.5 d_s/3.14 [d_s/(11.5*3.14) + 1]^{1.7} - 1$$

$$d_s = 0.81 \text{ m}$$

RUN 3

$$1.5/2.34 = 5.5 d_s/2.34 [d_s/(11.5*2.34) + 1]^{1.7} - 1$$

$$d_s = 0.67 \text{ m}$$

RUN 6

$$1.5/1.09 = 5.5 d_s/1.09 [d_s/(11.5*1.09) + 1]^{1.7} - 1$$

$$d_s = 0.44 \text{ m}$$



#### 5.2.4 Other scour formulas

##### Blench (1965)

$$d_s/D = 3.64/(d_{35})^{0.125} Fr^{0.5} (B/D)^{0.25} - 1$$

RUN 1

$$d_s = 1.5 * [3.64 / (17.5)^{0.125} (0.429)^{0.5} (1.5/3.14)^{0.25} - 1] = 0.58m$$

RUN 3

$$d_s = 1.5 * [3.64 / (17.5)^{0.125} (0.530)^{0.5} (1.5/2.34)^{0.25} - 1] = 0.99m$$

RUN 6

$$d_s = 1.5 * [3.64 / (17.5)^{0.125} (0.820)^{0.5} (1.5/1.09)^{0.25} - 1] = 1.59m$$

##### Breusers (1965)

$$d_s/D = 1.4 (B/D)$$

RUN 1

$$d_s = 1.5 * [1.4 (1.5/3.14)] = 1.00 m$$

RUN 3

$$d_s = 1.5 * [1.4 (1.5/2.34)] = 1.35 m$$

RUN 6

$$d_s = 1.5 * [1.4 (1.5/1.09)] = 2.89 m$$

Coleman (1971)

$$d_s = 1.49 B^{0.9} [u_m^2 / (2g)]^{0.1}$$

RUN 1

$$d_s = 1.49 * 1.5^{0.9} [2.38^2 / (2g)]^{0.1} = 1.90 \text{ M}$$

RUN 3

$$d_s = 1.49 * 1.5^{0.9} [2.54^2 / (2g)]^{0.1} = 1.92 \text{ M}$$

RUN 6

$$d_s = 1.49 * 1.5^{0.9} [2.68^2 / (2g)]^{0.1} = 1.94 \text{ M}$$

Hancu (1971)

$$d_s/D = 2.42 \text{ Fr}^{0.66} (B/D)^{0.66}$$

RUN 1

$$d_s = 1.5 [2.42 * (0.429)^{0.66} (1.5/3.14)^{0.66}] = 1.28 \text{ m}$$

RUN 3

$$d_s = 1.5 [2.42 * (0.530)^{0.66} (1.5/2.34)^{0.66}] = 1.78 \text{ m}$$

RUN 6

$$d_s = 1.5 [2.42 * (0.820)^{0.66} (1.5/1.09)^{0.66}] = 3.93 \text{ m}$$

Larras (1963)

$$d_s = (1.05 K) / B^{0.25} (B/D)$$

where

K = a multiplying factor which accounts for pier shape  
and angle of approach.

RUN 1

$$d_s = [1.05 (1)] / 1.5^{0.25} (1.5/3.14) = 0.45 \text{ m}$$

RUN 3

$$d_s = [1.05 (1)] / 1.5^{0.25} (1.5/2.34) = 0.61 \text{ m}$$

RUN 6

$$d_s = [1.05 (1)] / 1.5^{0.25} (1.5/1.09) = 1.31 \text{ m}$$

U.S. Geological Survey (1975)

$$d_s = 1.2 B^{0.8} \quad \text{for } d_{50} > 8 \text{ mm}$$

RUN 1

$$d_s = 1.2 (1.5)^{0.8} = 1.66 \text{ m}$$

RUN 3

$$d_s = 1.2 (1.5)^{0.8} = 1.66 \text{ m}$$

RUN 6

$$d_s = 1.2 (1.5)^{0.8} = 1.66 \text{ m}$$

### 5.2.5 New Scour Formula

The constants  $C_1$  and  $C_2$  will be determined by a two parameter least squares method to fit the data of Davoren's (Runs 1, 3, and 6). For each of these runs, the horizontal velocity profile is computed using the three parameter momentum based velocity distribution derived in this research. The effective depth of horizontal velocity is obtained from Equations (4-89) and (4-90). The specific energy components within the effective depth are then determined for the computed profile. The results are tabulated below in Table 6 where  $y_s$  = the observed scour.

| RUN           | 1     | 3     | 6     |
|---------------|-------|-------|-------|
| $u_m$ (m/s)   | 2.38  | 2.54  | 2.68  |
| $u_D$ (m/s)   | 2.80  | 3.20  | 3.10  |
| D (m)         | 3.14  | 2.34  | 1.09  |
| $y_s$ (m)     | 1.28  | 0.88  | 0.26  |
| $y_e$ (m)     | 0.90  | 0.83  | 0.59  |
| Velocity head | 0.181 | 0.186 | 0.303 |

TABLE 6

The coefficients computed are  $C_1 = 1.793$  and  $C_2 = -2.514$  and therefore the new scour equation is

$$d_s = 1.793 y_e - \left\{ 2.514 \int_0^{y_e} u^3 dy \right\} / \left\{ 2 g \int_0^{y_e} u dy \right\} \quad (5-1)$$

where

$d_s$  = the maximum depth of local scour,

$y_e$  = the effective depth of vertical velocity at the pier,

$u$  = the horizontal velocity at a depth  $y$  above the bed,

and

$g$  = the acceleration due to gravity.

This is the equation to be used for estimation of scour at the site of Davoren's (1985) data collection (site specific equation). This equation gives the following results for Runs used in the calibration.

RUN 1

$$d_s = 1.793(0.901) - 2.514(.181) = 1.16 \text{ m}$$

RUN 3

$$d_s = 1.793(0.826) - 2.514(.186) = 1.01 \text{ m}$$

RUN 6

$$d_s = 1.793(0.566) - 2.514(.303) = 0.25 \text{ m}$$

### 5.2.6 Remarks and conclusions

The previously presented assortment of scour formulas give a vast array of solutions (see TABLE 7). It can be seen that some of the formulas are very limited because of the variables used. These formulas, such as the U.S. Geological Survey formula, are not very robust as they do not account for the great variability in the scour process.

TABLE 7

| FORMULA        | CALCULATED SCOUR (m) |       |       |
|----------------|----------------------|-------|-------|
|                | RUN 1                | RUN 3 | RUN 6 |
| Shen           | 4.75                 | 4.49  | 3.61  |
| Laursen        | 0.81                 | 0.67  | 0.44  |
| Blench         | 0.58                 | 0.99  | 1.59  |
| Breusers       | 1.00                 | 1.35  | 2.89  |
| Coleman        | 1.90                 | 1.92  | 1.94  |
| Hancu          | 1.28                 | 1.78  | 3.93  |
| Larras         | 0.45                 | 0.61  | 1.31  |
| U.S.G.S.       | 1.66                 | 1.66  | 1.66  |
| EQU. (5-1)     | 1.16                 | 1.01  | 0.26  |
| Observed scour | 1.28                 | 0.88  | 0.26  |

Other formulas, such as Hancu's formula, are dominated by the effects of the velocity. These formulas tend to show that the scour is greater for higher values of velocity. As is shown by the actual field observations of Davoren (1985), this is not necessarily the case. In the Ohau river the scour was not proportional to the velocity, or Froude number, only.

Formulas that used the depth of flow as the major parameter causing scour gave better results than the other types. These formulas, such as Laursen's formula, still did not agree very well with the field data. They generally gave results that were too high.

The new scour formula presented here gave the best results when compared to the actual scour observation. This is not surprising since its coefficients  $C_1$  and  $C_2$  were calibrated using the field data. This shows that the general form of the scour equation presented is very good for estimating the scour, but the coefficients  $C_1$  and  $C_2$  are very site specific. It should also be remembered that this formula is based on a live-bed and it does not take into account any armoring of the channel that may occur. As armoring of the channel will greatly reduce the local scour, the lack of an armoring parameter in this formula is conservative.

## CHAPTER 6

### APPLICATION AT A PIER

#### 6.1 Introduction

To apply the risk analysis algorithm in its entirety, this study will use the data provided by Davoren (1985) in Runs 1 through 6. A wide range of simulated flow conditions were available in his study because the pier was placed downstream of the Ohau A hydro-electric power station. This provides a good range of flows to use in the risk analysis algorithm. Also the flows from the power station were steady for many hours so that problems arising from flood waves were not present.

The data in Runs 1, 3, and 6 were used in the calibration of the coefficients  $C_1$  and  $C_2$  of the scour equation. It should be noted that the recurrence interval of the flows of Runs 1, 3, and 6 were 33.0, 6.9, and 3.4 years respectively. The procedure will be applied to test the fit of the remaining data, Runs 2, 4, and 5. The recurrence interval of Run 2 is 100 years. The estimation of the maximum scour from the flow of this Run will be of particular interest.



The data for Runs 2, 4, and 5 that is provided by Davoren (1985) includes the mean velocity,  $u_m$ , the maximum scour depth,  $y_s$ , the discharge,  $Q$ , and the recurrence interval,  $T_r$ . A full horizontal velocity profile was not taken and the maximum velocity at the surface was not measured. To use the proposed scour method, the maximum velocity,  $u_D$ , will be estimated by means of the Prandtl - von Karman velocity equation. This equation gave good results near the surface of the flow.

## 6.2 Determination of flood PDF

The flood PDF that will be used in the application of the maximum scour algorithm is that given by Davoren (1985) and tabulated below in Table 8. This was obtained by Davoren from a Gumbel frequency analysis using 55 years of record. These frequencies are plotted and presented in Figure 14.

| Run | Date    | Recurrence interval<br>(years) | Discharge<br>(m <sup>3</sup> /s) |
|-----|---------|--------------------------------|----------------------------------|
| 1   | 9/20/82 | 33.0                           | 480                              |
| 2   | 9/22/82 | 100                            | 600                              |
| 3   | 10/6/82 | 6.9                            | 360                              |
| 4   | 1/19/82 | 2.75                           | 280                              |
| 5   | 1/20/82 | 1.2                            | 180                              |
| 6   | 8/09/82 | 3.4                            | 300                              |

TABLE 8

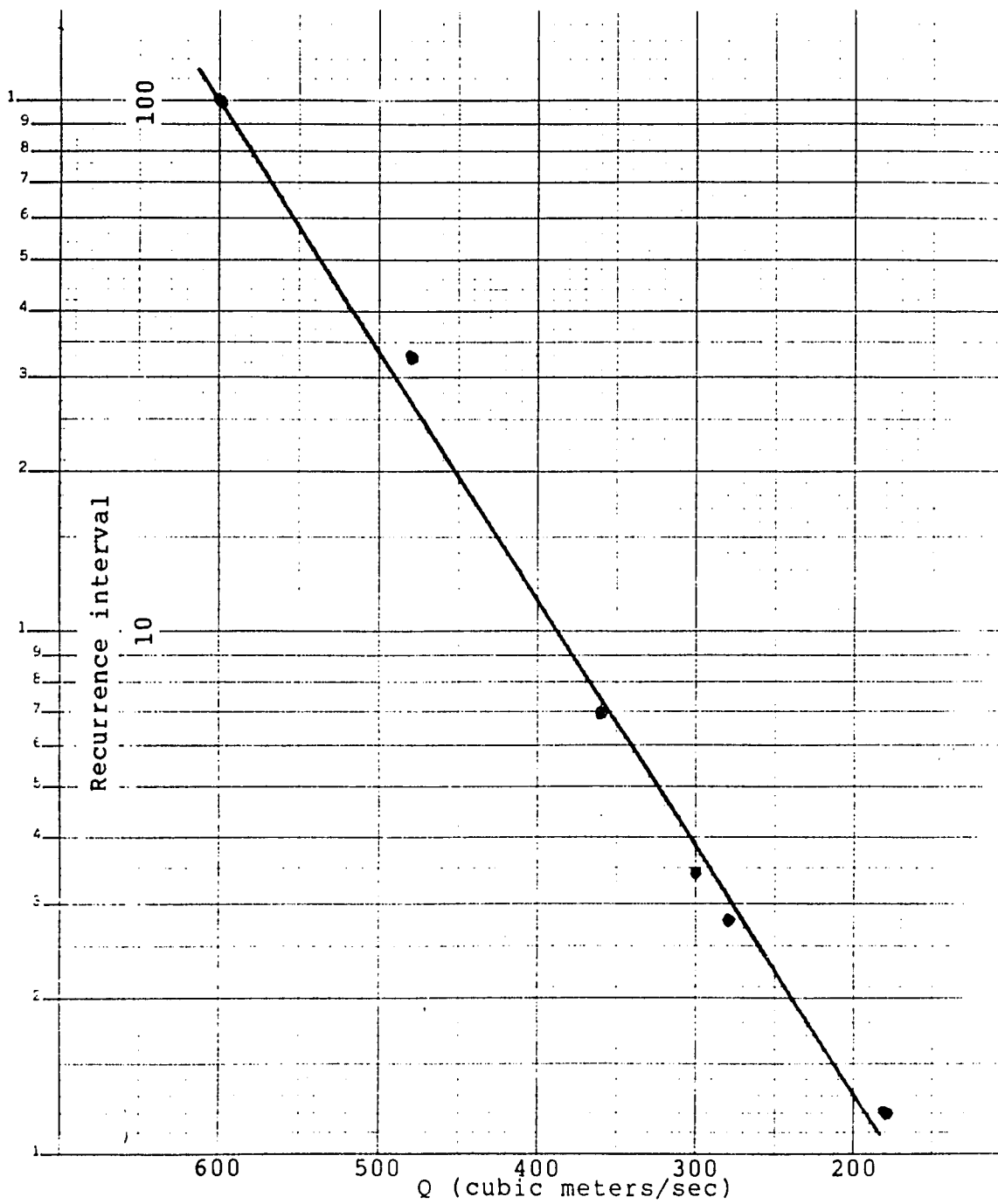


FIGURE 14

### 6.3 Determination of velocity distribution

The velocity distributions for Runs 1 through 6 are obtained using the three parameter momentum based entropy distribution Equation (4-29). The values of the coefficients  $A$ ,  $L_2$ , and  $L_3$  are tabulated below in Table 9 and the tabulation of each profile for Runs 1 through 6 are given in Appendix B. Velocities given below are in m/s and depth is in m.

| RUN   | 1       | 2       | 3       | 4       | 5       | 6       |
|-------|---------|---------|---------|---------|---------|---------|
| $u_m$ | 2.38    | 2.69    | 2.54    | 2.65    | 2.43    | 2.68    |
| $u_D$ | 2.80    | 3.70*   | 3.20    | 3.35*   | 3.10*   | 3.10    |
| $D$   | 3.14    | 3.02    | 2.34    | 1.44    | 1.32    | 1.09    |
| $A$   | -5.1504 | -2.6708 | -3.6423 | -3.6307 | -3.4058 | -5.8933 |
| $L_2$ | 2.1050  | 0.6271  | 1.2043  | 1.1282  | 1.1561  | 2.1471  |
| $L_3$ | .00612  | .00327  | .00528  | .00473  | .00574  | .00429  |

TABLE 9

\* Denotes velocities not observed by Davoren. They were computed from Prandtl - von Karman velocity equation.

#### 6.4 Determination of the scour and the scour PDF

First, the effective depth of vertical velocity is obtained for Runs 1 through 6 by using Equations (4-89) or (4-90). The scour is then computed using Equation (5-1). In this way, a scour value for each of the recurrence intervals given for Runs 1 through 6 is obtained. The results of these computations are tabulated below in Table 10 and the resulting scour PDF is plotted in Figure 15.

| RUN   | 1     | 2     | 3     | 4     | 5     | 6     |
|-------|-------|-------|-------|-------|-------|-------|
| $u_m$ | 2.38  | 2.69  | 2.54  | 2.65  | 2.43  | 2.68  |
| $u_D$ | 2.80  | 3.70* | 3.20  | 3.35* | 3.10* | 3.10  |
| D     | 3.14  | 3.02  | 2.34  | 1.44  | 1.32  | 1.09  |
| $y_e$ | 0.901 | 0.894 | 0.826 | 0.657 | 0.627 | 0.566 |
| $V_e$ | 0.180 | 0.127 | 0.186 | 0.241 | 0.205 | 0.304 |
| $d_s$ | 1.16  | 1.28  | 1.01  | 0.57  | 0.61  | 0.25  |

TABLE 10

\* Denotes velocities not observed by Davoren. They were computed from Prandtl - von Karman velocity equation.

$V_e$  denotes the effective velocity head.

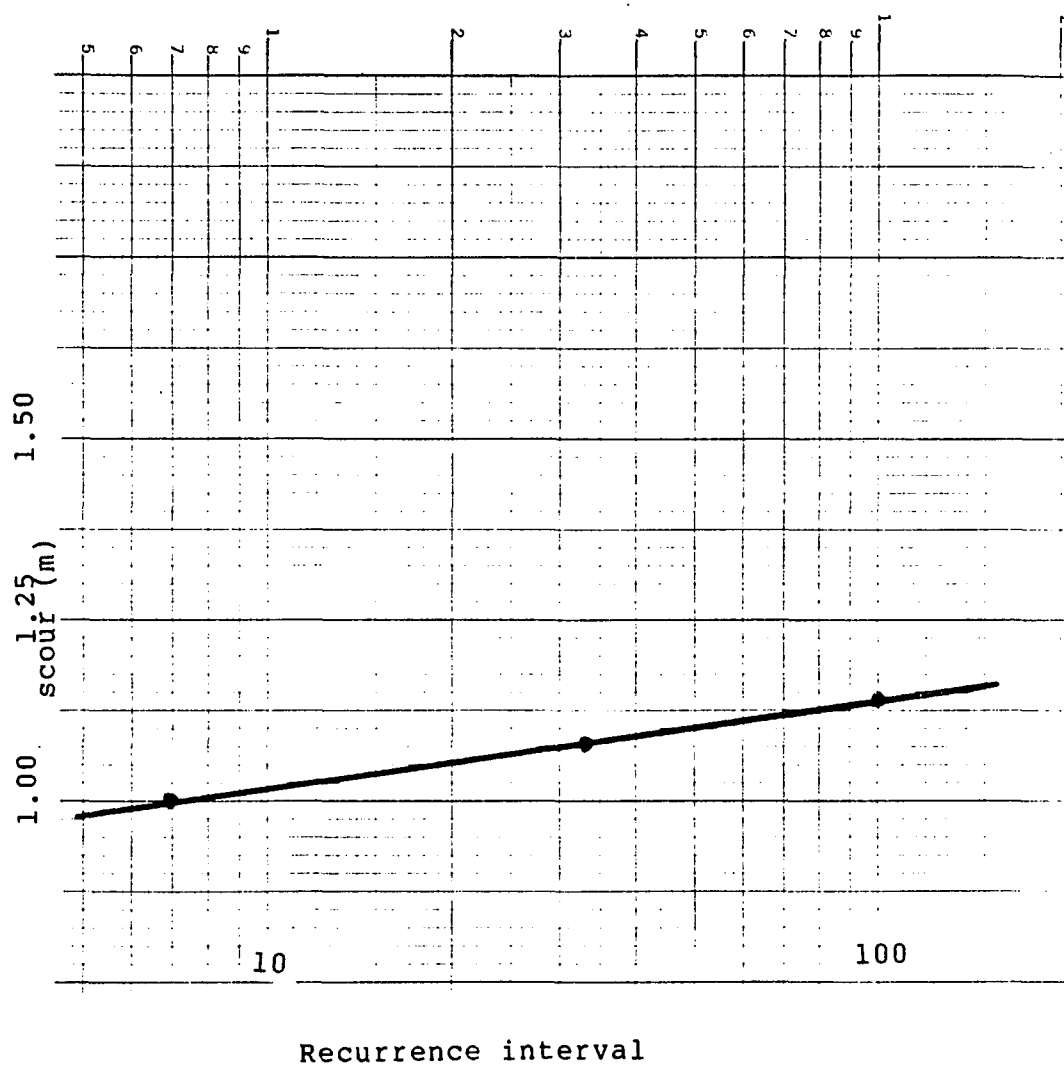


FIGURE 15

### 6.5 Comparison to actual data

The results of the developed scour formula are compared to Davoren's actual observed scour data below in Table 11.

| Run | Computed<br>Scour (m) | Observed<br>Scour (m) |
|-----|-----------------------|-----------------------|
| 1   | 1.16                  | 1.28                  |
| 2   | 1.28                  | 1.20                  |
| 3   | 1.01                  | 0.88                  |
| 4   | 0.57                  | 0.72                  |
| 5   | 0.61                  | 0.59                  |
| 6   | 0.25                  | 0.26                  |

TABLE 11

Table 11 shows that the computed scour PDF compares well with that for the observed scour ( $R^2 = .924$ ). Runs 1, 3, and 6 were used in the derivation of the scour formula but runs 2, 4, and 5 were not. This indicates that the scour PDF obtained from selected flood events can be a good indicator of the entire scour PDF. Also note that the return interval of Run 2 is 100 years. The difference between the computed value and the observed value is about 7%.

## CHAPTER 7

### CONCLUSIONS

In this study, the flood PDF, the horizontal velocity distribution, published vertical downflow measurements, and published local scour data were used to develop a systematic approach to the problem of estimation of the maximum scour at a bridge pier. This was done by a three step methodology.

First, the strength of the horseshoe vortex was estimated by the definition of a parameter called the effective depth. This effective depth was obtained by evaluation of the vertical velocity. The effective depth was defined as the depth of the approach flow that contributes to scouring. As evidenced by the formation of a bow wave in front of the pier, some of the approach flow is deflected upward and therefore can not cause scour. The depth at which downflow begins was estimated empirically from actual field data.

The downflow in front of a bridge pier is an indication of the strength of the horse shoe vortex system which causes scour. The scour formula derived provides a new and useful way of describing the local scour in terms of the downflow in front of the pier.



Second, a new method for computing the velocity profile was introduced. This method, the three parameter momentum based entropy method, was used to compute the velocity profile within the effective depth.

The concept of entropy was used to develop a this new velocity distribution for open channel flow. All of the velocity distributions tested gave good results near the surface, but the new entropy method was best ( $R^2 = .998$  for Run 1) in describing the velocity near the channel bed. This indicates that the new entropy method is more appropriate for use in describing near bed processes such as local scour.

The entropy method produces a family of algorithms for obtaining the velocity distribution. The momentum based velocity distribution had the best fit to the observed data ( $R^2 = .998$ ). This was because the coefficient  $L_3$  is one order of magnitude smaller than  $L_4$ . Therefore, the Maclaurin series for  $L_3$  converged quicker than for  $L_4$ . Expanding the energy based distribution will increase its accuracy but at the cost of added complexity. As the error analysis for the Maclaurin series showed, the momentum based method should still have a smaller error. Also, Chiu (1988) found that the magnitude of  $L_4$  was  $10^{-5}$  for the data tested. The approximation that  $L_4 = 0$  is therefore expected to give good results.

The new entropy velocity distribution developed can be easily used for other hydrologic purposes. By measuring the mean velocity and the maximum velocity, estimates of the coefficients  $A$ ,  $L_2$ , and  $L_3$  can be obtained for the three parameter momentum based entropy velocity distribution. The energy and momentum can then be estimated by numerical integration of this velocity profile. Good estimates of a river's energy and momentum coefficients,  $\alpha$  and  $\beta$ , can therefore be obtained by iteration.

Third, the velocity profile that was computed was used to obtain the specific energy in the effective depth. A new scour formula was obtained by relating this specific energy to the local scour.

None of the existing scour formulas were adequate in estimating the local scour. Formulas that used the depth of flow as the major parameter causing scour gave better results than the other types but still were generally too high. The new scour formula derived gave the best results for estimating the local scour ( $R^2 = .924$ ).

Data from the published observations of scour at a pier in the Ohau river were used to develop the new scour formula. This scour formula is site specific. The limitations are:

- 1) The procedure stated is based on the field data for an isolated pier. The results apply to an isolated pier only where the flow pattern is independent of any neighboring piers or obstructions. If the distance between two isolated piers were steadily reduced, the scour holes from each pier would eventually influence the other. The scour holes would overlap and the flow pattern of the horse shoe vortex of one would greatly influence the downward velocity at the front of the other pier.
- 2) The velocity distributions are based on the condition of turbulent flow. In turbulent flow the velocity vector is not constant, it fluctuates with time. This study uses only the time averaged velocity.
- 3) The data used for the new scour formula were for a flow condition with a live-bed. With this live-bed flow condition, sediment size is not a parameter for local scour.
- 4) It is assumed that no armoring of the channel will occur. If armoring of a channel occurs, the local scour is significantly reduced.
- 5) The pier used by Davoren in his study was a cylindrical steel pier. No conclusions can be drawn for piers of any other shape or material type.

The computed scour PDF compared well with that for the observed data ( $R^2 = .924$ ). This indicates that the scour PDF

obtained from selected flood events can be a good indicator of the entire scour PDF. The estimated scour of 1.28 m for the 100 year recurrence interval flow compared impressively well to the observed value of 1.20 m (difference = 7%).

The method used to obtain a scour PDF gave good results and was easy to apply. This approach can be used to obtain the scour PDF for any inventory of bridge structures by obtaining data for a limited number of random flood events. In this way, the relative risk for each structure can be obtained and a priority system for detailed inspection established. The data needed for the evaluation of each site is as follows:

- 1) Vertical velocity measurements for several random flood events to define the effective depth versus approach depth relationship. These measurements are not easily obtained at actual bridge sites. Detailed flume experiments can be done that would give the effective depth relationship for various bridge pier shapes, materials, and width/depth ratios. Also, the effective depth could be obtained for various bed materials for the clear water case.

- 2) Field measurements would be needed for a minimum number of flood events ( three flood events with return intervals of between 2 and 20 years). For each event, measurements of the mean horizontal velocity, the maximum horizontal velocity, and

the maximum scour would be needed. Presently, scour measurements are taken well after a flood event has occurred. This data does not give a true picture of the actual maximum scour that occurred during the event. The above information would be more difficult to obtain than present methods but with this data and the methodology in this research, the accuracy of scour estimation could be greatly increased.

Also, the coefficients  $C_1$  and  $C_2$  in the scour equation can be thought of as loss coefficients. This is the proportion of the potential and kinetic energy that cause scour. Flume experiments may be able to provide this information and then field measurements would be greatly reduced.

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**APPENDIX A**  
**MORE SCOUR FORMULAS**

The following is a list of additional scour formulas developed by other researchers (Davoren, 1985).

Blench (1965)

$$d_s/D = 3.64/(d_{35})^{0.125} Fr^{0.5} (B/D)^{0.25} - 1$$

Breusers (1965)

$$d_s/D = 1.4 (B/D)$$

Coleman (1971)

$$d_s = 1.49 B^{0.9} [u_m^2 / (2g)]^{0.1}$$

Hancu (1971)

$$d_s/D = 2.42 Fr^{0.66} (B/D)^{0.66}$$

Larras (1963)

$$d_s = (1.05 K)/B^{0.25} (B/D)$$

where

K = a multiplying factor which accounts for pier shape  
and angle of approach.

Laursen and Toch (1956)

$$d_s = 1.5 K B^{0.7} D^{0.3}$$

where

K = a multiplying factor which accounts for pier alignment and shape.

Subhash (1981)

$$d_s/B = 1.84 [D / B]^{0.3} (F_c)^{0.25} \quad (2-26)$$

U.S. Geological Survey (1975)

$$d_s = 1.2 B^{0.8} \quad \text{for } d_{50} > 8 \text{ mm}$$

## **APPENDIX B**

Velocity profiles

## Prandtl - von Karman Logarithmic Velocity Distribution

Data from Run 1

| DEPTH (m) | VELOCITY (m/s)                |      |      |
|-----------|-------------------------------|------|------|
|           | von Karman universal constant |      |      |
|           | 0.2                           | 0.3  | 0.4  |
| 0.05      | -                             | -    | -    |
| 0.10      | -                             | -    | -    |
| 0.15      | -                             | -    | -    |
| 0.20      | -                             | -    | -    |
| 0.25      | -                             | -    | 0.20 |
| 0.30      | -                             | -    | 0.38 |
| 0.35      | -                             | -    | 0.54 |
| 0.40      | -                             | -    | 0.68 |
| 0.45      | -                             | 0.13 | 0.80 |
| 0.50      | -                             | 0.28 | 0.91 |
| 0.55      | -                             | 0.41 | 1.01 |
| 0.60      | -                             | 0.53 | 1.10 |
| 0.65      | -                             | 0.64 | 1.18 |
| 0.70      | -                             | 0.74 | 1.26 |
| 0.75      | -                             | 0.84 | 1.33 |
| 0.80      | -                             | 0.92 | 1.39 |
| 0.85      | 0.11                          | 1.01 | 1.46 |
| 0.90      | 0.23                          | 1.09 | 1.51 |



## Prandtl - von Karman Logarithmic Velocity Distribution

(continued)

|      |      |      |      |
|------|------|------|------|
| 0.95 | 0.34 | 1.16 | 1.57 |
| 1.00 | 0.45 | 1.23 | 1.62 |
| 1.05 | 0.55 | 1.30 | 1.67 |
| 1.10 | 0.64 | 1.36 | 1.72 |
| 1.15 | 0.73 | 1.42 | 1.77 |
| 1.20 | 0.82 | 1.48 | 1.18 |
| 1.25 | 0.90 | 1.54 | 1.85 |
| 1.30 | 0.99 | 1.59 | 1.89 |
| 1.35 | 1.06 | 1.64 | 1.93 |
| 1.40 | 1.14 | 1.69 | 1.97 |
| 1.45 | 1.21 | 1.74 | 2.00 |
| 1.50 | 1.28 | 1.79 | 2.04 |
| 1.55 | 1.35 | 1.83 | 2.07 |
| 1.60 | 1.41 | 1.87 | 2.10 |
| 1.65 | 1.48 | 1.92 | 2.14 |
| 1.70 | 1.54 | 1.96 | 2.17 |
| 1.75 | 1.60 | 2.00 | 2.20 |
| 1.80 | 1.65 | 2.04 | 2.23 |
| 1.85 | 1.71 | 2.07 | 2.26 |
| 1.90 | 1.77 | 2.11 | 2.28 |
| 1.95 | 1.82 | 2.15 | 2.31 |
| 2.00 | 1.87 | 2.18 | 2.34 |
| 2.05 | 1.92 | 2.21 | 2.36 |

## Prandtl - von Karman Logarithmic Velocity Distribution

(continued)

|      |      |      |      |
|------|------|------|------|
| 2.10 | 1.97 | 2.25 | 2.39 |
| 2.15 | 2.02 | 2.28 | 2.41 |
| 2.20 | 2.07 | 2.31 | 2.43 |
| 2.25 | 2.11 | 2.34 | 2.46 |
| 2.30 | 2.16 | 2.37 | 2.48 |
| 2.35 | 2.20 | 2.40 | 2.50 |
| 2.40 | 2.25 | 2.43 | 2.52 |
| 2.45 | 2.29 | 2.46 | 2.55 |
| 2.50 | 2.33 | 2.49 | 2.57 |
| 2.55 | 2.37 | 2.51 | 2.59 |
| 2.60 | 2.41 | 2.54 | 2.61 |
| 2.65 | 2.45 | 2.57 | 2.63 |
| 2.70 | 2.49 | 2.59 | 2.64 |
| 2.75 | 2.53 | 2.62 | 2.66 |
| 2.80 | 2.56 | 2.64 | 2.68 |
| 2.85 | 2.60 | 2.67 | 2.70 |
| 2.90 | 2.64 | 2.69 | 2.72 |
| 2.95 | 2.67 | 2.71 | 2.74 |
| 3.00 | 2.71 | 2.74 | 2.75 |
| 3.05 | 2.74 | 2.76 | 2.77 |
| 3.10 | 2.77 | 2.78 | 2.79 |

## Power Law Velocity Distribution

Data from Run 1

| DEPTH (m) | VELOCITY (m/s) |      |
|-----------|----------------|------|
|           | coefficient n  |      |
|           | 6              | 7    |
| 0.05      | 1.40           | 1.55 |
| 0.10      | 1.58           | 1.71 |
| 0.15      | 1.69           | 1.81 |
| 0.20      | 1.80           | 1.89 |
| 0.25      | 1.84           | 1.95 |
| 0.30      | 1.89           | 2.00 |
| 0.35      | 1.94           | 2.05 |
| 0.40      | 1.99           | 2.09 |
| 0.45      | 2.03           | 2.12 |
| 0.50      | 2.06           | 2.15 |
| 0.55      | 2.09           | 2.18 |
| 0.60      | 2.13           | 2.21 |
| 0.65      | 2.15           | 2.24 |
| 0.70      | 2.18           | 2.26 |
| 0.75      | 2.21           | 2.28 |
| 0.80      | 2.23           | 2.30 |
| 0.85      | 2.25           | 2.32 |
| 0.90      | 2.27           | 2.34 |
| 0.95      | 2.29           | 2.36 |

## Power Law Velocity Distribution

(continued)

|      |      |      |
|------|------|------|
| 1.00 | 2.31 | 2.38 |
| 1.05 | 2.33 | 2.39 |
| 1.10 | 2.35 | 2.41 |
| 1.15 | 2.37 | 2.43 |
| 1.20 | 2.39 | 2.44 |
| 1.25 | 2.40 | 2.45 |
| 1.30 | 2.42 | 2.47 |
| 1.35 | 2.43 | 2.48 |
| 1.40 | 2.45 | 2.49 |
| 1.45 | 2.46 | 2.51 |
| 1.50 | 2.48 | 2.52 |
| 1.55 | 2.49 | 2.53 |
| 1.60 | 2.50 | 2.54 |
| 1.65 | 2.52 | 2.55 |
| 1.70 | 2.53 | 2.57 |
| 1.75 | 2.54 | 2.58 |
| 1.80 | 2.55 | 2.59 |
| 1.85 | 2.56 | 2.60 |
| 1.90 | 2.58 | 2.61 |
| 1.95 | 2.59 | 2.62 |
| 2.00 | 2.60 | 2.63 |
| 2.05 | 2.61 | 2.63 |
| 2.10 | 2.62 | 2.64 |

## Power Law Velocity Distribution

(continued)

|      |      |      |
|------|------|------|
| 2.15 | 2.63 | 2.65 |
| 2.20 | 2.64 | 2.66 |
| 2.25 | 2.65 | 2.67 |
| 2.30 | 2.66 | 2.68 |
| 2.35 | 2.67 | 2.68 |
| 2.40 | 2.68 | 2.69 |
| 2.45 | 2.69 | 2.70 |
| 2.50 | 2.70 | 2.71 |
| 2.55 | 2.70 | 2.72 |
| 2.60 | 2.71 | 2.73 |
| 2.65 | 2.72 | 2.73 |
| 2.70 | 2.73 | 2.74 |
| 2.75 | 2.74 | 2.75 |
| 2.80 | 2.75 | 2.75 |
| 2.85 | 2.76 | 2.76 |
| 2.90 | 2.76 | 2.77 |
| 2.95 | 2.77 | 2.78 |
| 3.00 | 2.78 | 2.78 |
| 3.05 | 2.79 | 2.79 |
| 3.10 | 2.79 | 2.79 |

Two Parameter Entropy Velocity Distribution  
and  
Three Parameter Energy Based Entropy Velocity Distribution  
Data from Run 1

| DEPTH (m) | VELOCITY (m/s) |               |
|-----------|----------------|---------------|
|           | 2 - Parameter  | 3 - Parameter |
| 0.05      | 1.07           | 1.17          |
| 0.10      | 1.36           | 1.42          |
| 0.15      | 1.52           | 1.57          |
| 0.20      | 1.64           | 1.68          |
| 0.25      | 1.73           | 1.76          |
| 0.30      | 1.81           | 1.83          |
| 0.35      | 1.87           | 1.89          |
| 0.40      | 1.93           | 1.94          |
| 0.45      | 1.98           | 1.99          |
| 0.50      | 2.02           | 2.03          |
| 0.55      | 2.06           | 2.07          |
| 0.60      | 2.10           | 2.10          |
| 0.65      | 2.13           | 2.13          |
| 0.70      | 2.17           | 2.16          |
| 0.75      | 2.19           | 2.19          |
| 0.80      | 2.22           | 2.22          |
| 0.85      | 2.25           | 2.24          |
| 0.90      | 2.27           | 2.26          |

## 2 and 3 Parameter Entropy Velocity Distributions

(continued)

|      |      |      |
|------|------|------|
| 0.95 | 2.29 | 2.29 |
| 1.00 | 2.32 | 2.31 |
| 1.05 | 2.34 | 2.33 |
| 1.10 | 2.36 | 2.35 |
| 1.15 | 2.38 | 2.36 |
| 1.20 | 2.39 | 2.38 |
| 1.25 | 2.41 | 2.40 |
| 1.30 | 2.43 | 2.42 |
| 1.35 | 2.44 | 2.43 |
| 1.40 | 2.46 | 2.45 |
| 1.45 | 2.47 | 2.46 |
| 1.50 | 2.49 | 2.48 |
| 1.55 | 2.50 | 2.49 |
| 1.60 | 2.51 | 2.50 |
| 1.65 | 2.53 | 2.52 |
| 1.70 | 2.54 | 2.53 |
| 1.75 | 2.55 | 2.54 |
| 1.80 | 2.56 | 2.55 |
| 1.85 | 2.58 | 2.56 |
| 1.90 | 2.59 | 2.58 |
| 1.95 | 2.60 | 2.59 |
| 2.00 | 2.61 | 2.60 |
| 2.05 | 2.62 | 2.61 |

## 2 and 3 Parameter Entropy Velocity Distributions

(continued)

|      |      |      |
|------|------|------|
| 2.10 | 2.63 | 2.62 |
| 2.15 | 2.64 | 2.63 |
| 2.20 | 2.65 | 2.64 |
| 2.25 | 2.66 | 2.65 |
| 2.30 | 2.67 | 2.66 |
| 2.35 | 2.68 | 2.67 |
| 2.40 | 2.69 | 2.68 |
| 2.45 | 2.70 | 2.69 |
| 2.50 | 2.70 | 2.70 |
| 2.55 | 2.71 | 2.71 |
| 2.60 | 2.72 | 2.71 |
| 2.65 | 2.73 | 2.72 |
| 2.70 | 2.74 | 2.73 |
| 2.75 | 2.74 | 2.74 |
| 2.80 | 2.75 | 2.75 |
| 2.85 | 2.76 | 2.76 |
| 2.90 | 2.77 | 2.76 |
| 2.95 | 2.77 | 2.77 |
| 3.00 | 2.78 | 2.78 |
| 3.05 | 2.79 | 2.79 |
| 3.10 | 2.80 | 2.79 |



## Three Parameter Momentum Based Entropy Velocity Distribution

| DEPTH (m) | VELOCITY (m/s)       |      |      |      |      |      |
|-----------|----------------------|------|------|------|------|------|
|           | Data from Run Number |      |      |      |      |      |
|           | 1                    | 2    | 3    | 4    | 5    | 6    |
| 0.05      | 0.91                 | 0.22 | 0.57 | 0.80 | 0.73 | 1.67 |
| 0.10      | 1.20                 | 0.41 | 0.90 | 1.22 | 1.11 | 1.99 |
| 0.15      | 1.38                 | 0.59 | 1.14 | 1.50 | 1.38 | 2.18 |
| 0.20      | 1.51                 | 0.75 | 1.32 | 1.71 | 1.58 | 2.31 |
| 0.25      | 1.61                 | 0.89 | 1.47 | 1.88 | 1.75 | 2.41 |
| 0.30      | 1.69                 | 1.02 | 1.60 | 2.02 | 1.88 | 2.49 |
| 0.35      | 1.76                 | 1.14 | 1.71 | 2.14 | 2.00 | 2.56 |
| 0.40      | 1.83                 | 1.25 | 1.80 | 2.25 | 2.11 | 2.63 |
| 0.45      | 1.88                 | 1.36 | 1.89 | 2.35 | 2.20 | 2.68 |
| 0.50      | 1.93                 | 1.46 | 1.97 | 2.43 | 2.28 | 2.73 |
| 0.55      | 1.97                 | 1.55 | 2.04 | 2.51 | 2.36 | 2.77 |
| 0.60      | 2.01                 | 1.64 | 2.11 | 2.58 | 2.43 | 2.81 |
| 0.65      | 2.05                 | 1.72 | 2.17 | 2.65 | 2.49 | 2.85 |
| 0.70      | 2.09                 | 1.80 | 2.22 | 2.71 | 2.55 | 2.88 |
| 0.75      | 2.12                 | 1.87 | 2.28 | 2.77 | 2.61 | 2.92 |
| 0.80      | 2.15                 | 1.94 | 2.33 | 2.82 | 2.66 | 2.94 |
| 0.85      | 2.18                 | 2.01 | 2.37 | 2.87 | 2.71 | 2.97 |
| 0.90      | 2.20                 | 2.08 | 2.42 | 2.92 | 2.75 | 3.00 |
| 0.95      | 2.23                 | 2.14 | 2.46 | 2.97 | 2.80 | 3.02 |
| 1.00      | 2.25                 | 2.20 | 2.50 | 3.01 | 2.84 | 3.05 |

## 3 Parameter Momentum Based Entropy Velocity Distributions

(continued)

|      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|
| 1.05 | 2.27 | 2.26 | 2.54 | 3.05 | 2.88 | 3.07 |
| 1.10 | 2.30 | 2.32 | 2.57 | 3.09 | 2.92 |      |
| 1.15 | 2.32 | 2.37 | 2.61 | 3.13 | 2.96 |      |
| 1.20 | 2.34 | 2.42 | 2.64 | 3.16 | 2.99 |      |
| 1.25 | 2.36 | 2.47 | 2.67 | 3.20 | 3.02 |      |
| 1.30 | 2.37 | 2.52 | 2.70 | 3.23 | 3.06 |      |
| 1.35 | 2.39 | 2.57 | 2.73 | 3.26 |      |      |
| 1.40 | 2.41 | 2.61 | 2.76 | 3.29 |      |      |
| 1.45 | 2.42 | 2.66 | 2.79 |      |      |      |
| 1.50 | 2.44 | 2.70 | 2.82 |      |      |      |
| 1.55 | 2.46 | 2.75 | 2.84 |      |      |      |
| 1.60 | 2.47 | 2.79 | 2.87 |      |      |      |
| 1.65 | 2.48 | 2.83 | 2.89 |      |      |      |
| 1.70 | 2.50 | 2.87 | 2.92 |      |      |      |
| 1.75 | 2.51 | 2.90 | 2.94 |      |      |      |
| 1.80 | 2.53 | 2.94 | 2.96 |      |      |      |
| 1.85 | 2.54 | 2.98 | 2.98 |      |      |      |
| 1.90 | 2.55 | 3.01 | 3.00 |      |      |      |
| 1.95 | 2.56 | 3.05 | 3.03 |      |      |      |
| 2.00 | 2.57 | 3.08 | 3.05 |      |      |      |
| 2.05 | 2.59 | 3.12 | 3.06 |      |      |      |
| 2.10 | 2.60 | 3.15 | 3.08 |      |      |      |
| 2.15 | 2.61 | 3.18 | 3.10 |      |      |      |

## 3 Parameter Momentum Based Entropy Velocity Distributions

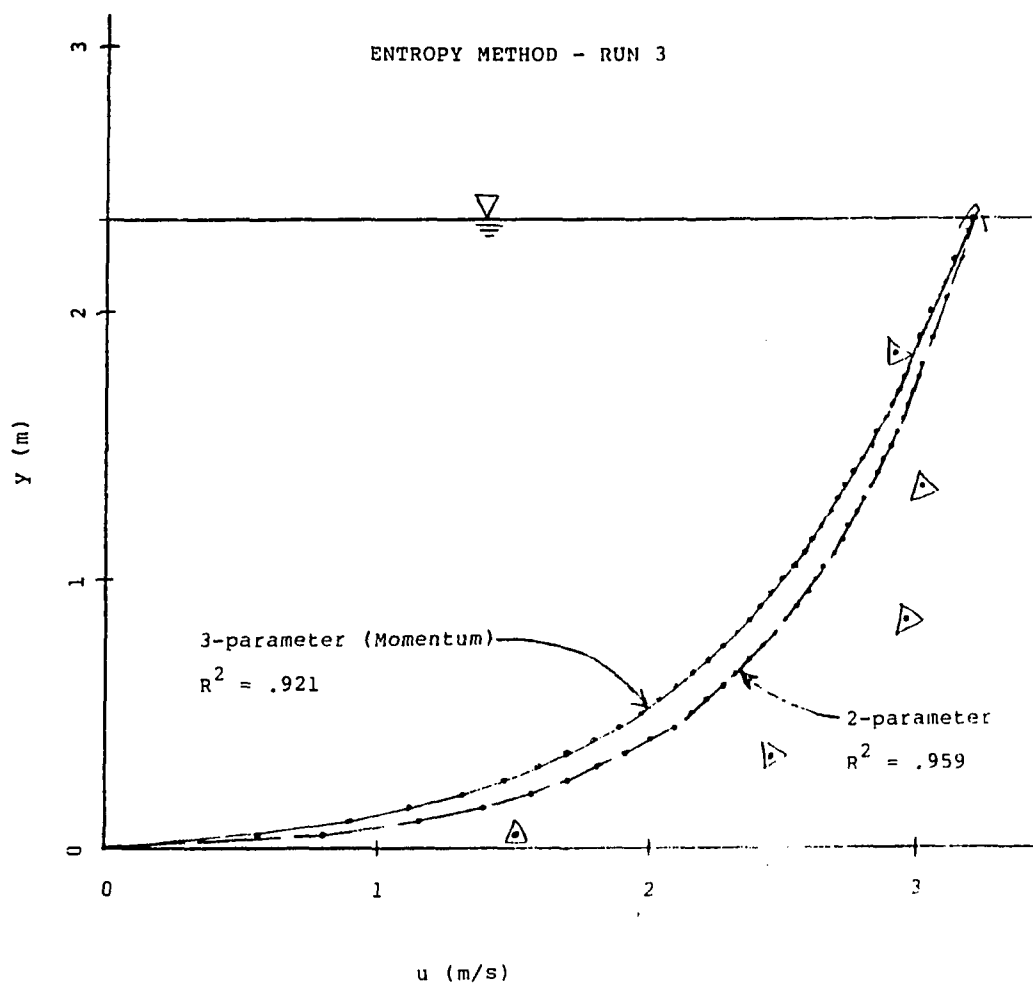
(continued)

|      |      |      |      |
|------|------|------|------|
| 2.20 | 2.62 | 3.21 | 3.12 |
| 2.25 | 2.63 | 3.24 | 3.14 |
| 2.30 | 2.64 | 3.27 | 3.16 |
| 2.35 | 2.65 | 3.30 |      |
| 2.40 | 2.66 | 3.33 |      |
| 2.45 | 2.67 | 3.36 |      |
| 2.50 | 2.68 | 3.39 |      |
| 2.55 | 2.69 | 3.41 |      |
| 2.60 | 2.70 | 3.44 |      |
| 2.65 | 2.71 | 3.47 |      |
| 2.70 | 2.72 | 3.49 |      |
| 2.75 | 2.72 | 3.52 |      |
| 2.80 | 2.73 | 3.54 |      |
| 2.85 | 2.74 | 3.57 |      |
| 2.90 | 2.75 | 3.59 |      |
| 2.95 | 2.76 | 3.62 |      |
| 3.00 | 2.76 | 3.64 |      |
| 3.05 | 2.77 |      |      |
| 3.10 | 2.78 |      |      |

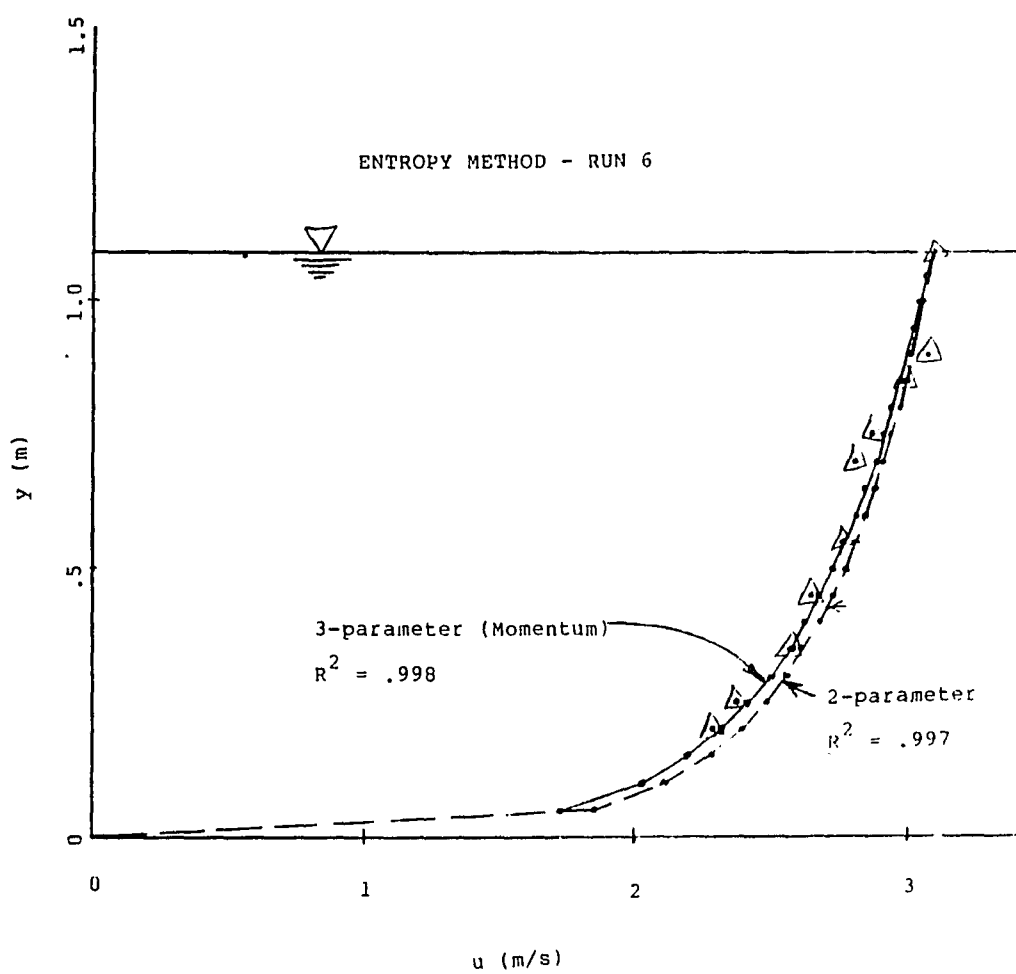
## **APPENDIX C**

Additional velocity profile comparisons

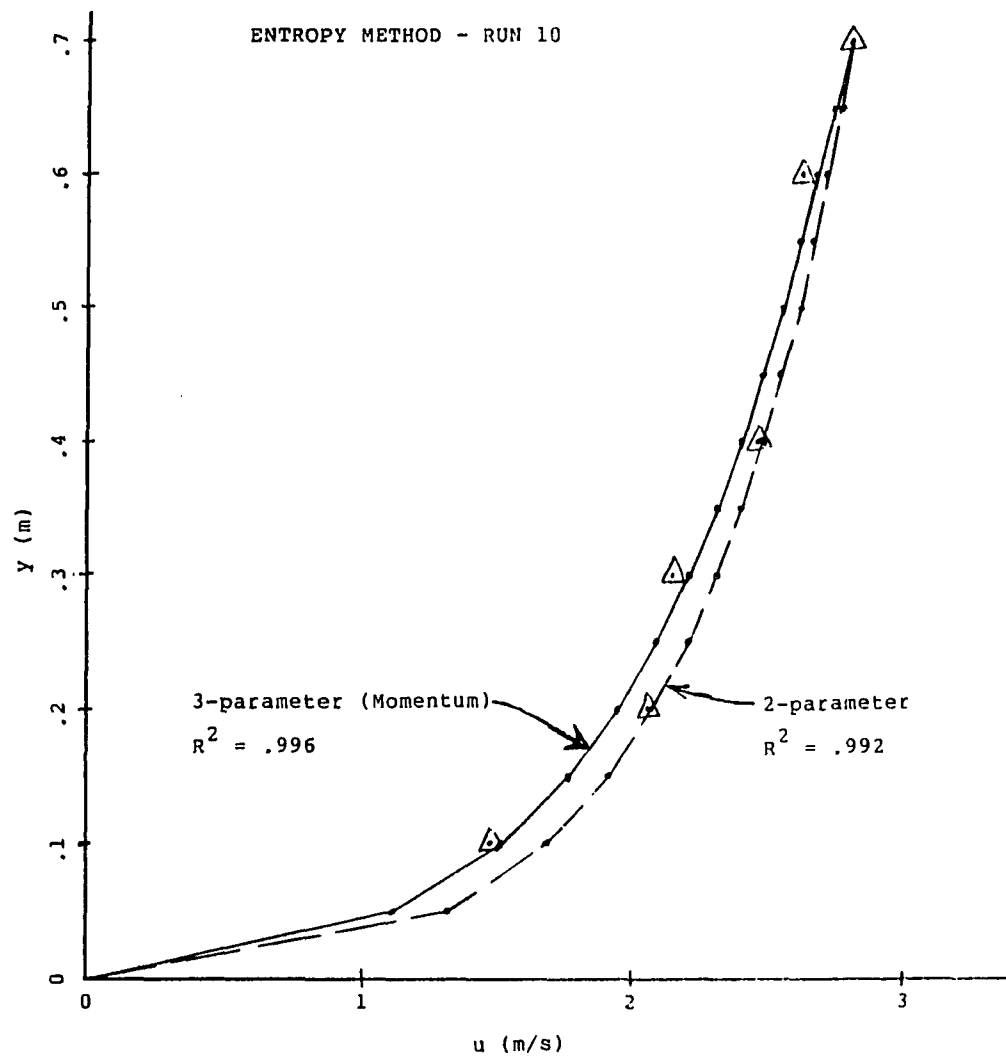
## RUN 3



## RUN 6



## RUN 10



## APPENDIX D

### Notations

$A = L_1 - 1$

$B$  = the pier diameter

$C_1$  and  $C_2$  = constants in the new scour equation

$D$  = the depth of flow in the channel

$d_s$  = the maximum scour depth

$E$  = the energy at a section

$F$  = Flow Froude number =  $u_m / (g D)^{1/2}$

$F_c$  = the critical froude number with respect to sediment motion.

$F_{\text{pier}}$  = the pier Froude number =  $u_m / (g B)^{1/2}$

$g$  = the acceleration due to gravity

$K$  = the von Karman universal constant which has a value of 0.40 for clear water and a value as low as 0.2 in flows with heavy sediment loads

$K_1 = M / (\rho D)$

$K_2 = 2g(E - D)$

$L_1, L_2, L_3$ , and  $L_4$  = Lagrange coefficients

$M$  = the momentum transferred across a section

$n$  = a parameter determined by the frictional resistance at the bed

$Q$  = the discharge

$Re_{\text{pier}}$  = the pier Reynolds number =  $u_m B / \mu$



$S$  = the channel slope

$T_r$  = recurrence interval

$u$  = the horizontal velocity at a distance  $y$  from the  
channel bed

$u_D$  = the maximum velocity of the flow which occurs at  
the surface of the flow ( $y = D$ )

$u_m$  = the mean approach velocity

$u_*$  = the shear velocity

$y_e$  = the effective depth of vertical velocity at the  
pier

$y_s$  = the maximum depth of local scour

$\mu$  = the viscosity

$\rho$  = the density

## **VITA**

Donald E. Barbe' received a B.S. degree in Mathematics from the University of New Orleans in 1970. He continued his studies at Tulane University and at the University of New Orleans and received a M.S. degree in Engineering at the University of New Orleans in 1986. He has been a practicing civil engineer for fifteen years and is registered in the states of Louisiana, Mississippi, and Texas. In the spring of 1987, he enrolled in the Graduate School at Louisiana State University to pursue a Doctor of Philosophy degree in Civil Engineering. He has been a Research Assistant in the Department of Civil Engineering. He is now a candidate for the Doctor of Philosophy degree in Civil Engineering.

DOCTORAL EXAMINATION AND DISSERTATION REPORT

Candidate: Donald Edward Barbe'

Major Field: Civil Engineering

Title of Dissertation: Probabilistic Analysis of Bridge Scour Using the  
Principle of Maximum Entropy

Approved:

James Cruise  
Major Professor and Chairman

F. Glen Hembry  
Dean of the Graduate School

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Date of Examination:

4/24/90